NON-LINEAR GROWTH OF TRAPPED PARTICLE MODES IN LINEARLY STABLE, CURRENT-CARRYING PLASMAS – A FUNDAMENTAL PROCESS IN PLASMA TURBULENCE AND ANOMALOUS TRANSPORT

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(Received 1 August 2005; Accepted in final form 22 November 2005)

Abstract. The two-stream instability as a fundamental process in a current-carrying plasma is reconsidered. Its well-established linear version, based on kinetic Landau theory, predicts a threshold for the drift velocity between both species below which the plasma should be stable. We report on simulations which, however, show that a plasma as a non-linearly responding medium can be destabilized well below this threshold. Responsible for this unexpected behaviour are coherent, electrostatic, trapped particle structures such as phase space vortices or holes which can grow non-linearly out of thermal noise receiving their energy from the net imbalance of loss of electron kinetic energy and gain of ion kinetic energy. The birth of predominantly zero-energy holes is shown numerically being associated with initial, non-topological fluctuations. The latter are not subject to Landau damping, as they lie outside the realm of linear wave theory. For a pair plasma a typical scenario is presented, which encompasses several regimes such as non-linear growth of multiple holes, saturation and fully developed structural turbulence as well as an asymptotic approach to a new collisionless equilibrium. During the transient, structural state the plasma transport appears to be highly anomalous.

Keywords: non-linear instability, phase space vortices, anomalous transport, structural turbulence, collisionless current-carrying plasmas

1. Introduction

In physical sciences – astro and plasma physics being no exceptions – a common consensus about the treatment of waves or fluctuations seems to exist which, when expressed in words, sounds so trivial that it is not even considered worth to be mentioned or commented explicitly in the literature including textbooks, monographs or whatever. The underlying concept, which everybody, being confronted with, immediately would accept can be formulated as follows:

> Small amplitude waves in the infinitesimal amplitude limit can be described and predicted by the linearized version of the governing equations. Only for larger amplitudes non-linear effects come into play.

This concept of dealing with waves in collective dynamical systems, which we may call the standard wave concept (SWC), is the foundation of all anomalous

Space Science Reviews (2005) 121: 313–331
DOI: 10.1007/s11214-006-5382-8

turbulence theories, bifurcation scenarios or studies on non-linear wave propagation, and it is very hard to find any reference which does not rely on this SWC. It is certainly meaningful and will work in situations where linear wave theory predicts a linear instability being driven internally by gradients, anisotropies, etc. or externally by beam injection, mean flow, wave heating, application of a voltage across a bounded plasma, etc. to mention some examples. We may, however, ask the question as to whether in cases of LINEAR STABILITY this implies that the system is then preferentially in a quiescent state of no or negligible wave activity. The answer, for which the present paper will offer concrete examples, is that even in such a linearly stable situation one is confronted with an appreciable amount of waves or fluctuations which have a profound effect on the plasma and will modify it completely in a manner which cannot be treated by SWC. Or, in other words, linear instability will be sufficient but not necessary for the prevalence of anomalous effects, as often found in astrophysical and plasma-physical environments. The problem, we are reconsidering, is the well-known two-stream instability, occurring in a current-carrying plasma, in its simplest geometry, namely one dimensional in the configuration space. Kinetic linear wave theory predicts a threshold $\mathcal{V}_\mathrm{D}^*$ for the drift velocity $\mathcal{V}_\mathrm{D}$ between electrons and ions, below which the plasma is linearly stable. We are mainly concerned with ideal collisionless plasmas but also weak collisional effects will be considered at the end, and specialize ourselves to linear stable, weakly driven plasmas for which $\mathcal{V}_\mathrm{D} < \mathcal{V}_\mathrm{D}^*$.

2. Related Previous Works

A first numerical hint that something goes wrong with linear wave analysis and associated SWC has been given by Dupree and co-workers (Berman *et al.*, 1985) who performed a particle in cell (PIC) simulation for a $m_i/m_e = 4$, $T_e/T_i = 1$ plasma with $\mathcal{V}_\mathrm{D} = 1.75 < \mathcal{V}_\mathrm{D}^* = 1.95$. In such a two-stream stable plasma structure formation in the electron and ion phase space took place arising from initially random fluctuations which are relatively high in such a numerical system. Both phase spaces after some initial transient state look rather turbulent and are characterized by filamentary and vortex-like structures, which intermittently intersperse the whole body of distributions, making any approach of this state by SWC obsolete.

Phase space vortices or holes, on the other hand, as non-linear, stationary solutions of the Vlasov–Poisson system has been described for a thermal Maxwellian plasma more than three decades ago by Schamel and co-workers (Schamel, 2000 and references therein). By application of the so-called potential method, a solution of the problem was presented which consists in two parts,

(i) in the pseudo-potential (Sagdeev-potential) $V(\phi)$, where ϕ is the normalized electrostatic potential satisfying without loss of generality $0 \leq \phi(x) \leq \psi$, and
(ii) in the non-linear dispersion relation (NDR), given by $V(\psi) = 0 = V(0)$.

Whereas $V(\phi)$ supplies information about the shape or spectral content of the non-linear wave, the NDR determines the wave speed in terms of the various external parameters such as $\theta = T_e/T_i$, $\delta = m_e/m_i$, $\mathcal{V}_D$ and ψ or internal parameters, such as the trapping parameters for electrons β and ions α. A number of new branches of wave solutions could be distinguished such as the slow electron or slow ion acoustic branch, which have no relation anymore to the known linear branches. In the weak amplitude limit $\psi \ll 1$, the corresponding density expression for each species has the typical form of a half-power expansion, such as Schamel (2000)

$$n(\phi) = 1 + a\phi + b\phi^{3/2} + \cdots \tag{1}$$

The coefficient a is typical of O(1) and reflects the contribution stemming from linear wave analysis. When the second coefficient b is O(1) too, then for sufficiently small perturbations the last term in (1) is negligible and the SWC turns out to be applicable (as for linear instabilities). In the present case of linear stability, however, b becomes $O(\psi^{-1/2}) \gg 1$ for a given wave speed because it has to include for consistency trapped particle effects. The latter fact originates from the excavation of the particle distribution at resonant velocity, $v = v_0$, which is a necessary condition for the existence of self-consistent equilibria. The third term in (1), hence, catches up with the second term and contributes at the same level than the linear term. This means that particle trapping constitutes a non-negligible ingredient of wave theory already at the lowest possible order, in contrast to the underlying assumption made for SWC. A dip in the distribution function introduces a region with an opposite slope, which, as we will see later, appears to be a triggering mechanism for hole excitation already at the extremely small (infinitesimal) wave amplitude limit. A distribution of this type is referred to in the following as a *non-topological* distribution, since the topology in velocity space, more specifically its slope, is altered locally in comparison with the unperturbed background distribution.

As a typical example of a trapped particle mode we mention the solitary electron hole (Schamel, 1979, 1986) propagating with v_0 in a thermal plasma within a background of immobile ions. In that case, the pseudo-potential $V(\phi)$ becomes (for $\psi \ll 1$)

$$-V(\phi) = \frac{8}{15} b\phi^2 [\sqrt{\psi} - \sqrt{\phi}] \tag{2}$$

representing a bell-shaped sech^4-potential $\phi(x)$, whereas the NDR reads

$$-\frac{1}{2} Z_r'(v_0/\sqrt{2}) = \frac{16}{15} b\sqrt{\psi} \tag{3}$$

where b is given by

$$b = \pi^{-1/2}\big(1 - \beta - v_0^2/2\big) \exp\big(-v_0^2/2\big) \tag{4}$$

and Z_r is the real part of the plasma dispersion function. From (2) we see that b must be positive for a solution to exist which implies in view of (3) and of the x-dependence of the $Z_r'(x)$-function that $0 < v_0 < 1.307$, which means that a solution

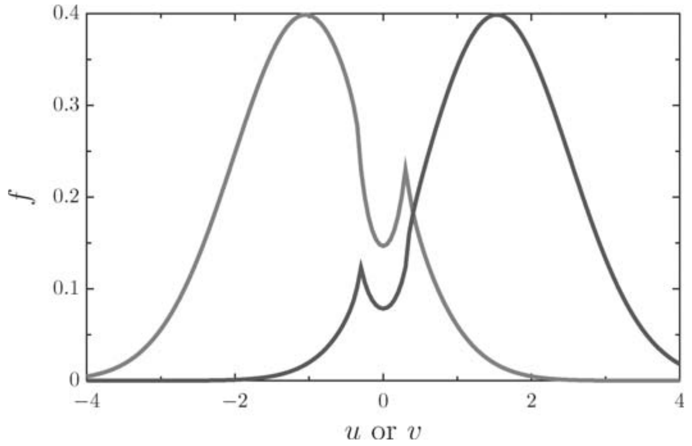

Figure 1. A typical electron (green) and ion (red) velocity distribution, respectively, for a solitary electron hole in the negative energy range of a current-carrying plasma.

only exists in the thermal bulk of the unperturbed distribution (where linear theory would predict strong Landau damping). Prescribing v_0 and hence the left-hand side of (3) we see that $b \propto \psi^{-1/2} \gg 1$ which, from (4), results in $-\beta \propto \psi^{-1/2} \gg 1$. The trapped electron parameter β, appearing in the trapped electron distribution function (Schamel, 2000), must hence be sufficiently negative for a solution to exist, implying a dip (depression) in the total distribution function at v_0.

More generally, a variety of solutions can be found in the three-dimensional parameter space spanned by the wavenumber k_0 and the two spectral parameters B_e and B_i, the latter two being related with θ, δ, α, β, ψ and $\mathcal{V}_D$, as shown in Luque *et al.* (2002), and Schamel and Luque (2005). Moreover, it is found (Griessmeier and Schamel, 2002; Griessmeier *et al.*, 2002) that a structure-carrying plasma can be in a lower energy state in comparison to that of the unperturbed plasma which means that Δw, denoting this difference, can be negative. We shall call a solitary hole with $\Delta w < 0$ ($\Delta w = 0$) a negative-energy (zero-energy) hole. A typical solution in phase space for a solitary electron hole of negative energy in a current-carrying plasma ($\mathcal{V}_D > 0$) is shown in Figure 1. It clearly exhibits the hole character at resonant velocity, here in both distributions.

Furthermore, it is interesting to note that solitary ion hole solutions of negative energy exist in which $\mathcal{V}_D < \mathcal{V}_D^*$ for ANY temperature ratio θ [see e.g. Figure 4 of Griessmeier and Schamel (2002)].

3. Non-linear Growth of Holes in Ordinary Plasmas

For ordinary plasmas ($\delta = m_e/m_i \ll 1$) we performed numerical simulations to learn more about hole excitation and the role they are playing in the dynamics. First we repeated the PIC simulations of Berman *et al.* (1985) with an improved PIC code (10 times more particles, 2 times more grid points) to check whether the observations made by them survive in a code of higher resolution. We essentially confirmed their results and, in addition, found that both solitary electron and ion holes are generated near $\Delta w = 0$ in phase space spontaneously out of thermal noise, each structure experiencing growth in course of time. During growth, ion holes are decelerated, sliding into their $\Delta w_i > 0$ region, whereas electron holes are accelerated, coming into rest finally in their $\Delta w_e < 0$ region. Both events happen simultaneously without any violation of energy conservation, as $\Delta w = \Delta w_i + \Delta w_e \simeq 0$. The plasma, after the elapse of some time, appears to be rather structured and filamentary, despite the fact that no linear instability was active during the whole evolution. To get completely rid of particle discreteness and to show that similar results emerge also within the continuous, kinetic description, we applied a Fourier–Hermite expansion code to the Vlasov–Poisson system, as developed in Korn and Schamel (1996a,b). This code has a two-fold advantage: one can incorporate firstly initial conditions with high precision according to analytic theory and secondly one can artificially switch off non-linearity (through the removal of the $E\partial_v f_1$ term, where f_1 is the perturbed distribution function) to find out the differences between a linear and a non-linear code. Three different cases, taken from Luque and Schamel (2005), have been investigated in which holes play a fundamental role.

Firstly, we imposed as a starting condition a self-consistent electron hole of small amplitude ($\psi = 0.01 \ll 1$). Figure 2 shows the evolution within both codes.

The linear code, marked by LINEAR (blue, dotted line), yields non-surprisingly Landau damping and the overall decay of the structure. The non-linear code (NON-LINEAR; red, solid line), however, shows the persistence of the hole during the whole evolution without any attenuation or distortion in accordance with a stable

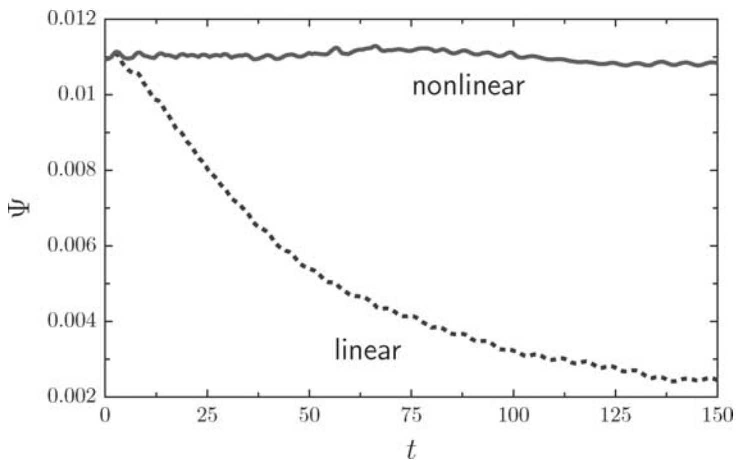

Figure 2. Time dependence of the amplitude of a stationary electron hole within the two codes.

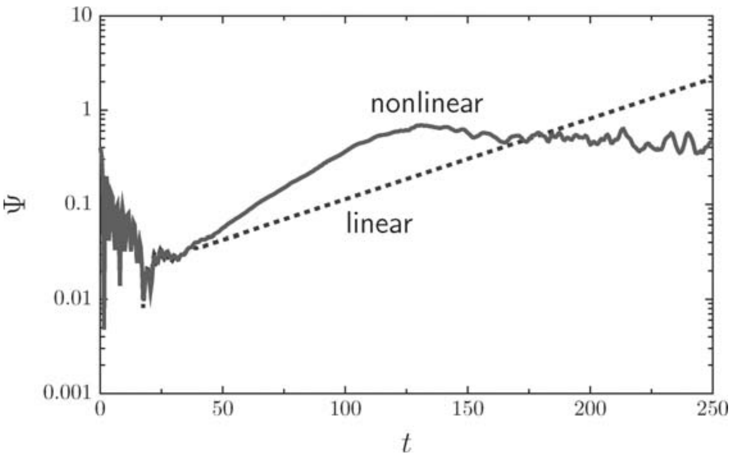

Figure 3. Time dependence of the maximum potential for the ordinary two-stream instability within the two codes.

equilibrium. A similar result has already been reported in Figure 2 of Korn and Schamel (1996a). Although $\psi \ll 1$, linear wave theory fails in describing the exact evolution.

Secondly, the ordinary two-stream instability was followed in Figure 3 with both codes ($\delta = 0.01, \theta = 1, \mathcal{V}_D = 2 > \mathcal{V}_D^* = 1.439$).

We have used five Fourier modes and 2000 Hermite polynomials and random noise initially. After a short period of adaption to self-consistency ($0 < t < 25$) the most growing mode establishes in both codes according to linear theory ($25 < t < 35$). Later the known differences in the evolution arise, a continuation of linear growth within the linear code and the non-linear growth and subsequent quenching by particle trapping and creation of phase space structures within the non-linear code. Everything in this linear instability regime develops as pretended by the SWC and would support its general validity if not opposite examples can be found.

Thirdly, we reinvestigated the so-called non-linear Landau scenario by imposing initially a distribution of the kind

$$f_e(x, v, t = 0) = (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}(v-\mathcal{V}_D)^2}[1 + \varepsilon \cos(kx)] \tag{5}$$

in which the velocity dependence of the perturbation is the same as that of the unperturbed shifted Maxwellian, having hence the same “topology”. The drift was chosen to lie in the linearly STABLE regime ($\mathcal{V}_D = 1$).

We see in Figure 4 no differences in the evolution between both codes up to $t \approx 40$, namely the linear Landau damping behaviour, mainly because the initial imposed perturbation was already very weak and satisfied its criterion. Nevertheless, after the decay of the wave amplitude of about 5 orders of magnitude surprisingly

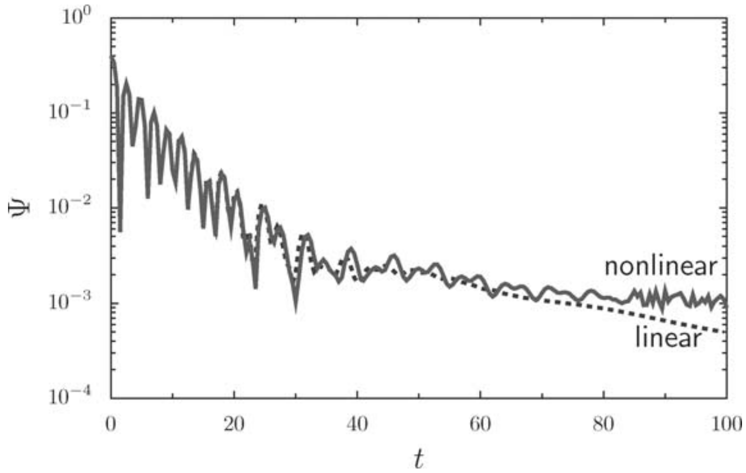

Figure 4. Time dependence of the maximum potential for the non-linear Landau damping scenario within the two codes.

a difference is seen between both codes. A tiny non-linear hole develops for large times in accord with an earlier result of Manfredi (1997). The explanation of this unexpected behaviour, we offer, is that the initial imposed fluctuation has decayed up to a level where non-topological random fluctuations, inherent in each code and plasma, became competitive and subsequently took over the leading role in the dynamical evolution triggering the birth of the tiny hole. Hence, even in the extremely low amplitude situation (for $50 < t$), non-linearity and particle trapping rule the evolution in contradiction to the assumptions made in the SWC. Although initially the condition for linear Landau damping, $| \partial_v f_1 | \ll | \partial_v f_0 |$, was satisfied and seemed to be satisfied even better in course of time, non-linearity came up inevitably by the emergence and subsequent dominance of non-topological fluctuations and the dynamics triggered by them. At least from here on the critical reader should be convinced that the SWC cannot claim general valitity and can fail under certain circumstances.

Finally, we performed further simulations (Luque and Schamel, 2005) (not shown here) where in a linearly two-stream stable situation a primary ion hole was imposed at $t = 0$ together with a small fluctuation in both trapping parameters which introduced a kind of detuning of the self-consistent coherent structure in a non-topological manner. We could see secondary hole structures emerging in course of time which were propagating and hence did not demand extra energy in accordance with the energy law. The energy fed into the secondary zero-energy structures stemmed from the imbalance of loss of electron kinetic energy and gain of ion kinetic energy. With this experience in mind, we are now prepared to study and to understand in a sense the non-linear evolution of a pair plasma found numerically in the linearly STABLE regime.

4. Non-Linearly Unstable Pair Plasmas

A pair plasma consists of two species with equal masses and temperatures and has therefore the advantage of a reduced parameter space, namely $\delta = 1, \theta = 1$. It is ubiquitously met in the universe in terms of an electron–positron plasma (Liang *et al.*, 1998; Miller and Wiita, 1998) and could also be created in laboratories in terms of a fullerene C_{60}^{+}, C_{60}^{-} plasma, e.g. in Oohara and Hatakeyama (2003). Its evolution is characterized by a single time scale instead of two, rather distinct time scales in case of an ordinary plasma which are due to the large mass disparity of its charge carriers. Responsible for the fact that both species acquire in local thermal equilibrium the same temperature is the symmetry in the momentum exchange during binary collisions. A simplification is found also theoretically, as the variety of analytic solutions in terms of trapped particle modes of the Vlasov–Poisson system is reduced and hence structure formation appears to be more transparent (Luque and Schamel, 2005). Shown here is a simulation with $\mathcal{V}_{\mathrm{D}} = 2 < \mathcal{V}_{\mathrm{D}}^{*} = 2.6$ together with 64 Fourier modes and 800 Hermite polynomials. For more details we refer to Luque *et al.* (2006). Initially incoherent, non-topological fluctuations f_1 were imposed which, albeit small, do not satisfy the applicability condition of linear wave theory $| \partial_v f_1 | \ll | \partial_v f_0 |$. As a footnote we mention that this was not necessary as a similar evolution with a delayed onset of non-linear instability could be seen if we had started with a lower noise level. This is in accord with the PIC simulations of Berman *et al.* (1985), the laboratory experiments of Moody and Driscoll (1995) or the coasting beam experiments at the Fermi Main Ring (Colestock and Spentzouris, 1996). In these Vlasov–Poisson systems hole generation in a linearly stable configuration, arising out of thermal noise, was observed at the “lowest measurable signal level”. The effect of tiny, non-topological fluctuations was merely to speed up the evolution and is, hence, of quantitative rather than of qualitative nature. Even if we had started with a topological fluctuation of the kind (5), as in the third example of the previous section (non-linear Landau damping), structure formation would have been seen unavoidably. (The reason why the hole seen for large times in this third example was stationary rather than growing was the absence of a drift.)

In Figure 5 the time evolution of the field energy w_{f} is plotted, as obtained from the linear code (blue, dotted line) and from the full non-linear code (red, solid line).

In the linear run we recognize the overall damping, as expected, although a complete disappearance of the fluctuations is not what is observed. Instead, the noise retains on a lower level and consists of fluctuations, which are not subject to Landau damping being either due to numerical effects (round off errors, discreteness effects, etc.) and/or to the initial component of non-topological fluctuations. (We note that for the applicability of the linear Landau damping scenario the initial fluctuations have to satisfy not only smallness but also smoothness (regularity) criteria, which are generally not met in a noisy initial spectrum of fluctuations. As this linear evolution contradicts essentially the following non-linear evolution, we

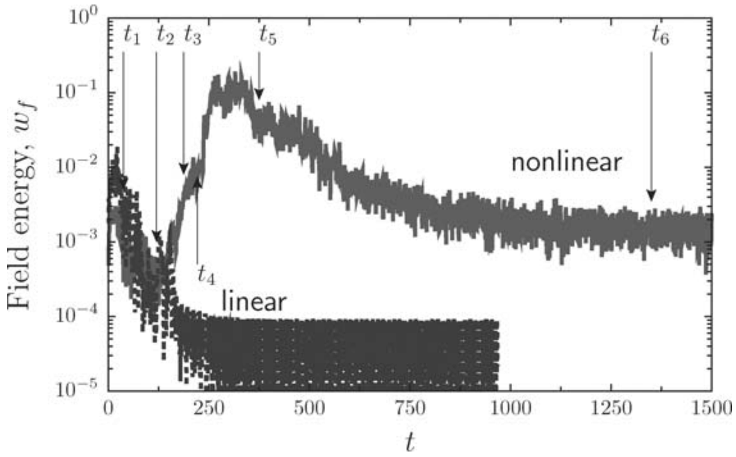

Figure 5. Time dependence of the field energy; linear run (blue, dotted line), non-linear run (red, solid line).

can state that any argument being based on the existence of linear modes and on their non-linear extensions, such as parametric instability, etc., will fail in describing and understanding of the non-linear evolution.)

In the non-linear evolution, our main concern, four distinct phases can be distinguished:

(1) an initial damping in $0 < t < 120$,
(2) an exponential growth in $120 < t < 260$,
(3) a saturation and a quasi-stationary state in $260 < t < 375$, and
(4) an approach towards a new, less energetic structural equilibrium for $375 < t$.

To learn, what happened internally in the plasma, we have made snapshots of both phase spaces and densities as well as of the potential at several characteristic time instants marked by arrows.

4.1. Damping Phase and Preparation of a Seed Hole

In Figure 6 a snapshot at $t_1 = 37.5$ belonging to the damping phase is presented.

It shows a lot of coherency in both phase spaces indicating an ongoing process of structure formation. Fluctuations in the initial noise that violate the linear Landau damping criterion and survive, as does the non-vanishing component in the linear run, trigger now the birth of coherent structures. As seen from the macroscopic potential and density plots, showing three coherent-like patterns, a kind of phase-locking process seems to be active during this phase, which terminates near $t \approx 120$, the time when w_f becomes minimum.

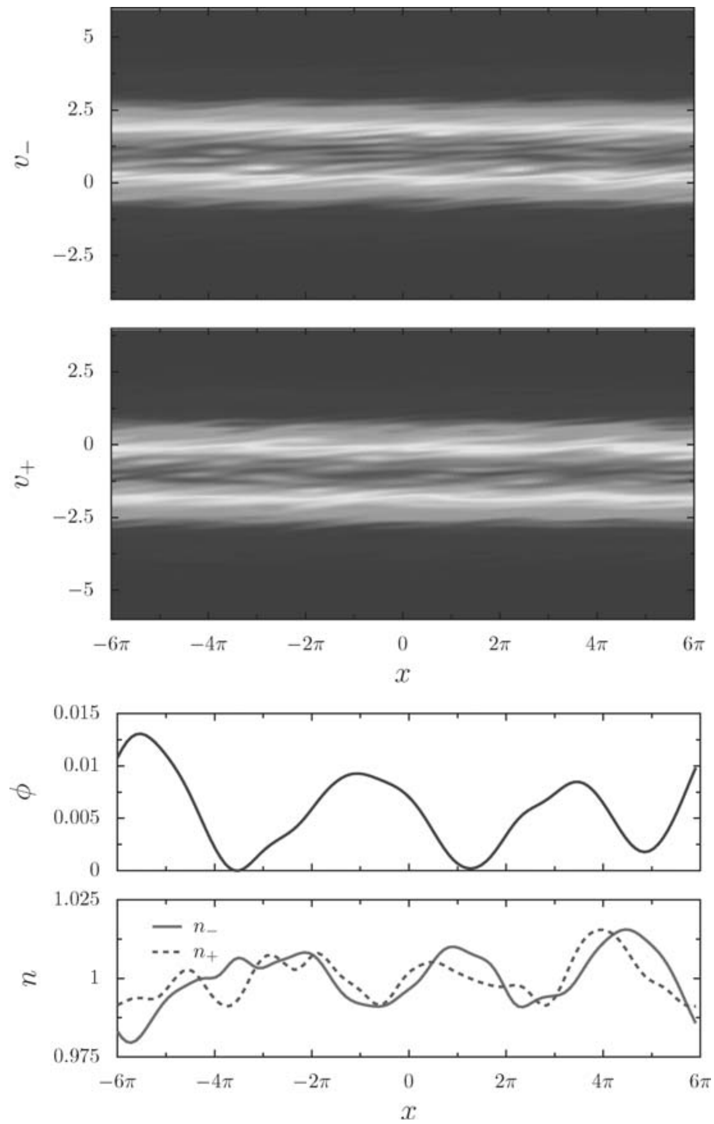

Figure 6. Snapshot of phase space configurations, potential and densities at $t = 37.5 =: t_1$.

4.2. Multiple Generation and Growth of Holes

At this time, $t_2 = 120$, a single hole in f_- at $x \approx -2\pi$ and $v_0 \approx 0$ has emerged together with some further wave activity at v_-, $v_+ \approx 0$, Figure 7.

This seed hole in the minus species – as best seen in the macroscopic plots – is exposed to a negative electric field, which is due to the slight asymmetry in the potential hump. The hole is hence non-stationary and experiences growth and

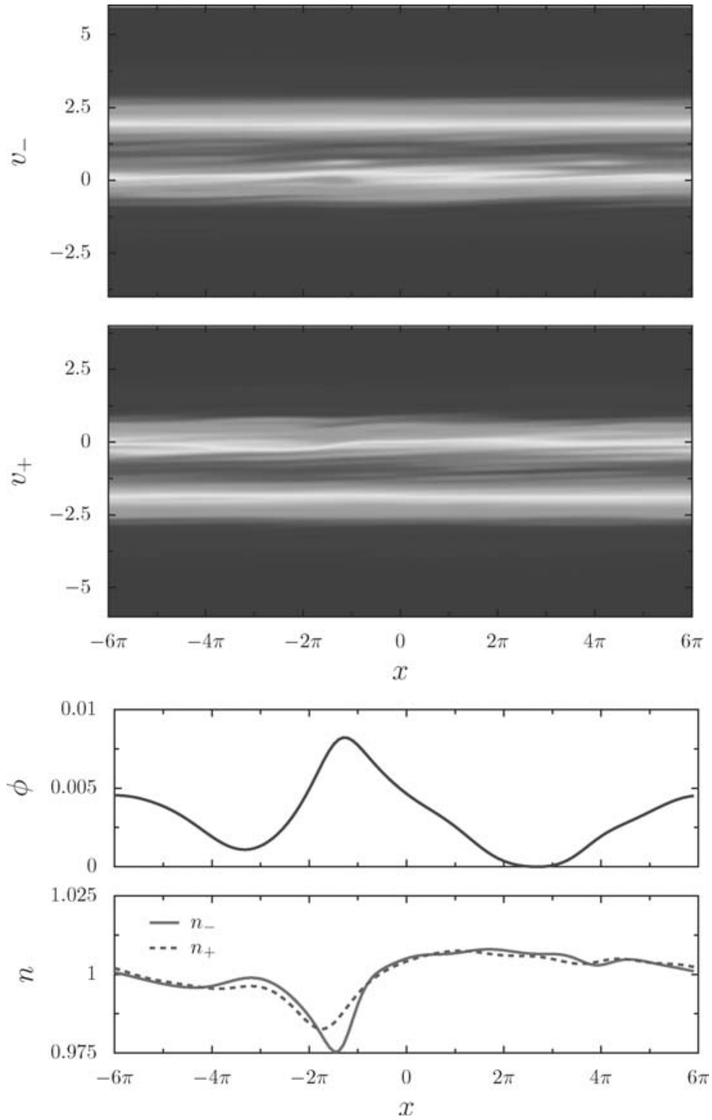

Figure 7. Snapshot of phase space configurations, potential and densities at $t = 120 =: t_2$.

acceleration, as the encircled structure in f_- of Figure 8, taken at $t_3 = 187.5$, indicates.

One plausible reason for this process is the encircled filament in the f_+ component, as explained in Luque *et al.* (2005), but also other plausibility arguments can be presented. One of them is that a −hole (a minus hole or a hole in the minus species), which can be interpreted as a macroparticle of positive charge Q, negative mass M (and position X) embedded in a – fluid, experiences an

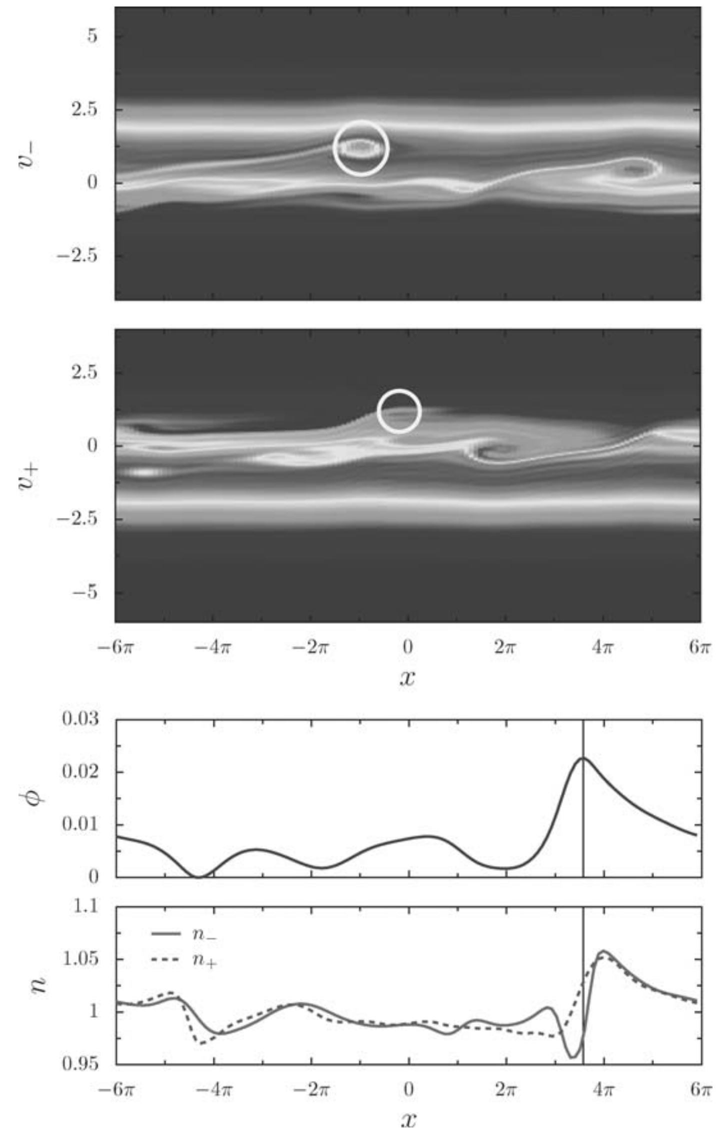

Figure 8. Snapshot of phase space configurations, potential and densities at $t = 187.5 := t_3$.

acceleration when $E < 0$, according to $M\ddot{X} = QE$. Here, Q and M are defined by $(Q, M) := \int dx \int dv(q_-, m_-)\tilde{f}_{-t}$ where $\tilde{f}_{-t} < 0$ represents the depression (hole) in the −distribution in the trapped range over which the v-integration is to be taken (Dupree, 1982; Schamel, 1986).

A similar observation about hole acceleration has been made by Eliasson and Shukla (2004) who found that a −hole is attracted (repelled) by a maximum (minimum) of n_+. This effect can be seen from the n_+-plot of Figure 7, noting again the slight offset of the potential hump with respect to the density minimum and the asymmetry of the latter.

Another explanation of the acceleration process during growth can be found from the non-linear theory of electron and ion holes (Bujarbarua and Schamel, 1981). In this work, the NLDs were investigated, Figures 3, 7 and 8, respectively, showing contours of constant phase velocity in the parameter space spanned by the amplitude ψ and the trapping parameter β and α, respectively. Assuming a nearly constant β (resp. α) an increase of ψ corresponds to a slowing down of the structure in a current-free plasma which amounts to an approach to the respective distribution centre. Transferred to our situation of a current-carrying plasma with shifted unperturbed Maxwellians this implies a speeding up of the $-$hole and a deceleration of the $+$hole. This latter property we have already observed in the PIC simulations and will be seen later again.

However, the most prominent feature in Figure 8 is seen near $x \approx 4\pi$ as a large bipolar trapping structure is created near $v_-, v_+ \approx 0$ in both species. In this velocity region Δw is approximately zero such that it does not cost extra energy for the plasma to build up the structure and achieves this by the aforementioned redistribution of energies. What is seen is the birth of a pair of holes, as this structure later on splits into a – hole and a $+$hole, as is seen in Figure 9, which is taken near the maximum of w_f, namely at $t_4 = 221.25$.

During growth the – hole appears to be accelerated and the $+$hole decelerated in accordance with the NLD. A similar splitting processes has been reported in Saeki and Genma (1998). At the same time, the birth of a new pair of holes (or even a triplet) emerges in region $x < 0$. This phase is characterised by the generation and amplification of holes accompanied by an energy transfer to electrostatic field energy w_f which grows exponentially fast and reaches maximum value at about $t = 260$.

4.3. Saturation and Fully Developed Structural Turbulence

Although the generation of new holes persists, w_f in this short phase is no longer increasing. Responsible for this saturation is a new process coming into play, namely an anomalous structural diffusion process which is activated by the coherent structures near $t = 260$ balancing the increase w_f due to structure formation.

Figure 10 shows a highly structured, filamentary state in both phase spaces near the end of this phase at $t = t_5 = 375$. The finite number of Hermite polynomials used in the code can no longer resolve the fine structures in phase space giving rise to a coarse-grained distribution. This introduces a kind of resistivity into the system. As shown and explained in Korn and Schamel (1996b) and Schamel and Korn (1996), a coherent structure increases the resistivity of a plasma provided the mobility of ions is taken into account in the code, a process which should be even more pronounced for a pair plasma. This diffusion

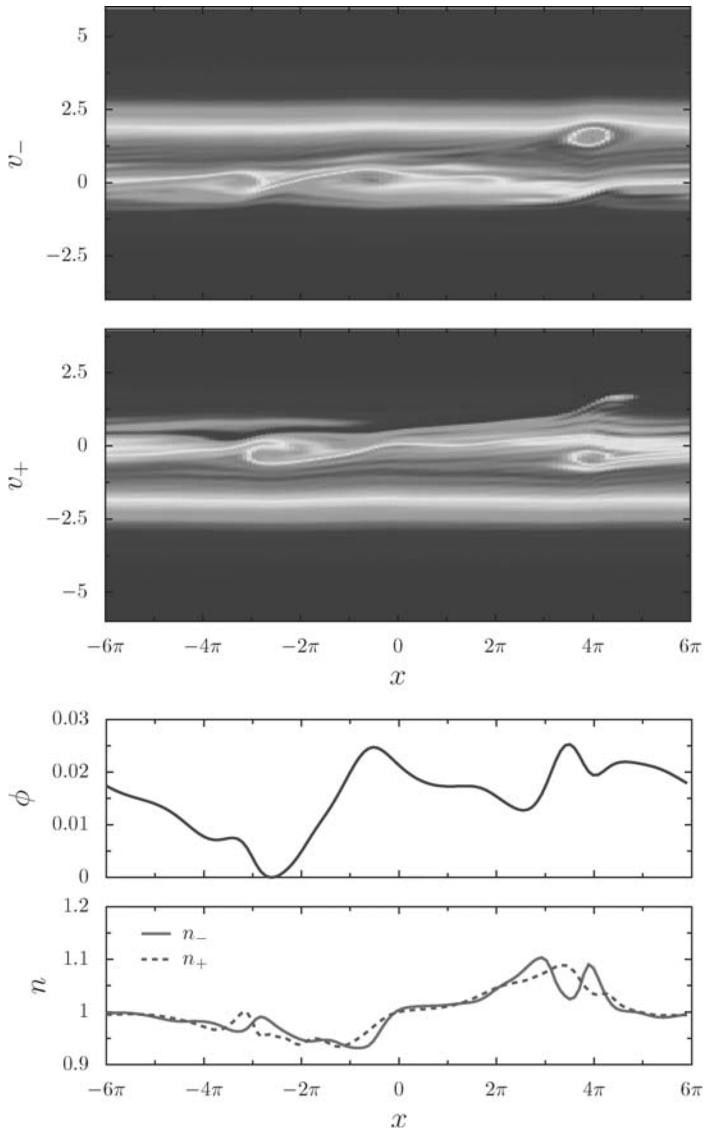

Figure 9. Snapshot of phase space configurations, potential and densities at $t = 221.25 =: t_4$.

affects the whole bulk of the distributions, heating up both species and reducing their drift velocities. This is shown in Figure 11, where the temporal behaviour of both mean velocities $\langle v \rangle_{+,-}$ and of the effective temperatures are plotted.

It is essentially this phase in which these macroscopic quantities experience a change.

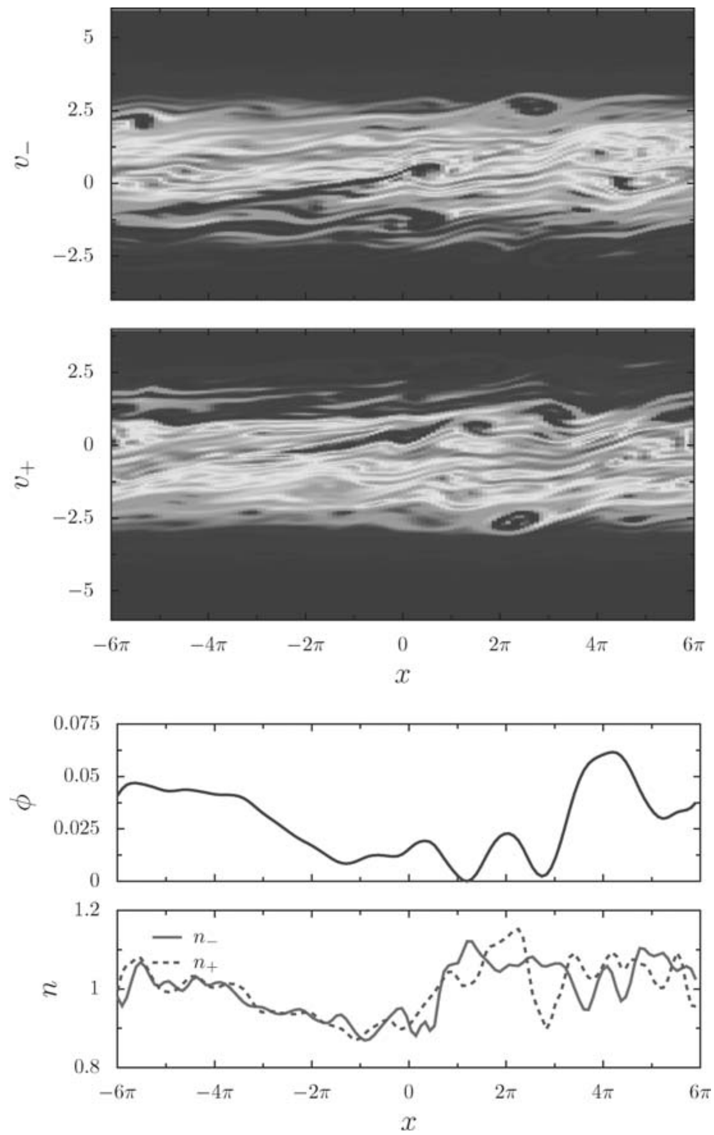

Figure 10. Snapshot of phase space configurations, potential and densities at $t = 375 =: t_5$.

4.4. Relaxation to a New Collisionless Equilibrium

In the last phase, when the birth of holes is quenched by the modifications of the distributions, the anomalous, structural diffusion, together with a merging and coalescence of phase space structures, eliminates all fine structures leaving few + holes in the system, as Figure 12, taken at $t = t_6 = 1350$, shows.

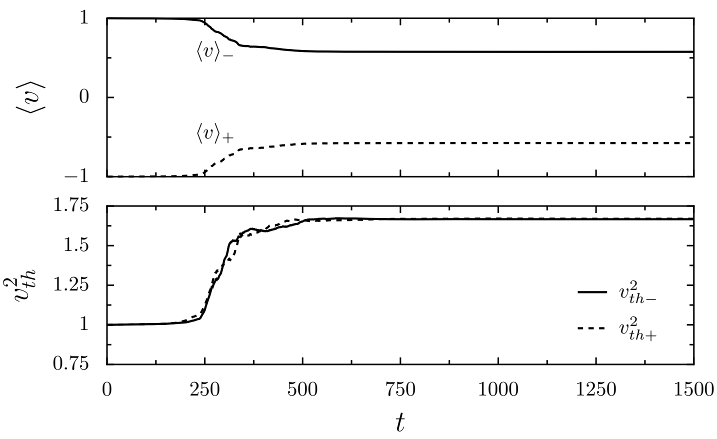

Figure 11. Time dependence of mean velocities and effective temperatures.

In this relaxation phase, the distributions become smooth again and are more or less flat-topped, except for f_+, which has an additional dip. This is seen in Figure 13, in which both distributions at $t = 1500$ are drawn.

The anomalous diffusion process is now quenched too. What survives is a new collisionless equilibrium state with flat-topped distributions and some few holes which finally coalesce into a single hole. That here a +hole finally survives in a non-symmetrical manner has its origin probably in the imposed initial data.

5. Summary and Conclusions

In this paper the role of non-linearity in a collisionless plasma, originating from electrostatic trapping, has been reinvestigated. A Vlasov–Poisson code in one dimension, which allows to switch off non-linearity to learn about its consequences, revealed that, whenever non-topological fluctuations are present in a noisy, initial spectrum, a current-carrying plasma becomes non-linearly unstable although the ambient drift velocity was chosen below the critical speed of linear instability. Responsible for this violation of the SWC, according to which no wave activity should arise, are trapped particle modes, such as electron and ion holes, which control the dynamics in each phase of the dynamical evolution.

In a typical scenario one first notices the preparation and growth of a seed hole, which is subsequently replaced by the multiple generation of holes or pairs of holes. In the exponentially growing phase of non-linear instability the growing structures are accelerated in case of electron holes and decelerated in case of ion holes. This stage is then followed by the onset of anomalous, structural diffusion which balances non-linear growth and leads to a phase of a highly structural but otherwise stationary turbulence. Time asymptotically, the chaotic state decays towards

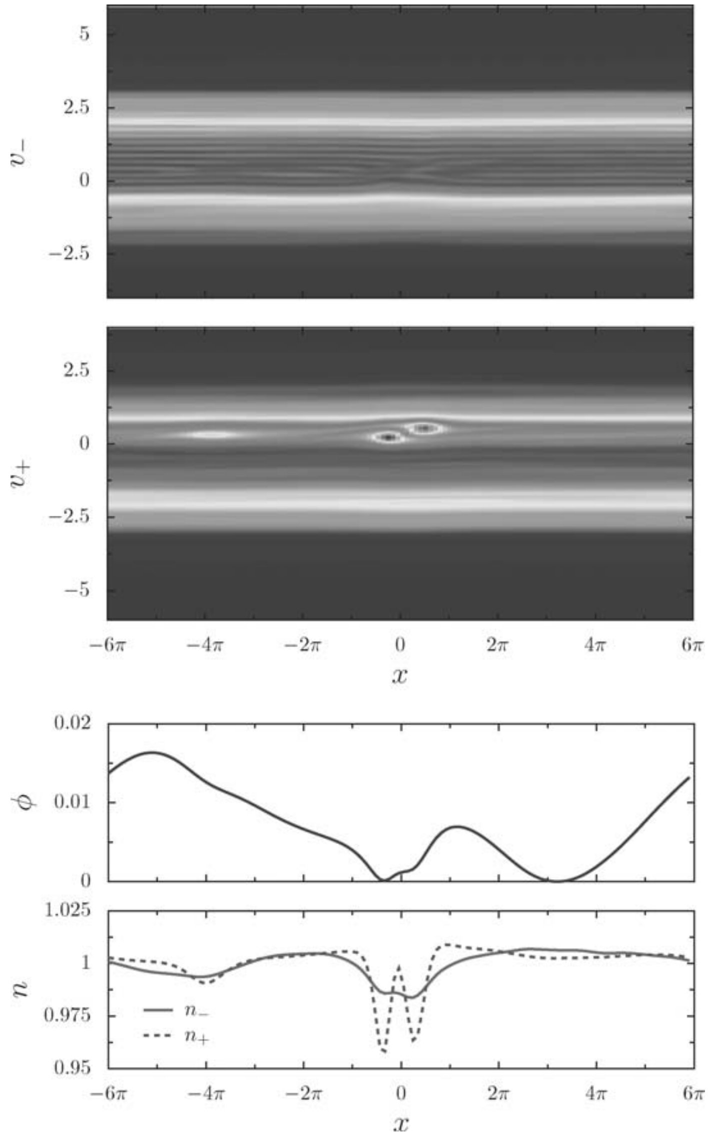

Figure 12. Snapshot of phase space configurations, potential and densities at $t = 1350 =: t_6$.

a new collisionless equilibrium which is characterized by flat-topped distributions on which few surviving holes are superimposed. This type of evolution, which was shown explicitly for a pair plasma, is expected to hold for an ordinary plasma as well with some changes in the involved time scales. It hence exhibits the dominant role trapping vortices are playing in 1D and prevails if a sufficiently strong magnetic field is present, preventing the decay of these structures into perpendicular modes.

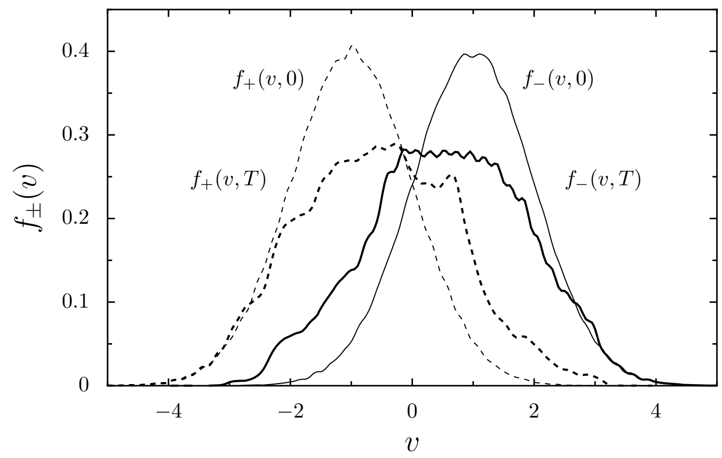

Figure 13. Velocity distribution functions at $t = 0$ and $t = 1500$, respectively.

A weakly collisional, fully ionised, magnetized, current-carrying plasma, such as met in space or in fusion devices, is hence subject temporarily to a non-linear growth of coherent perturbations before the new collisionless, structural equilibrium state is attained followed by the long-term approach to thermal equilibrium due to the remaining sparse collisions and associated classical transport processes.

This typical scenario reflects a fundamental process being active in ideal or nearly ideal plasmas for the following reasons:

(i) It is met universally, since residual, non-topological fluctuations will always be present in the superimposed noise.
(ii) Zero-energy phase space holes will always be latently present in a plasma being excited non-linearly.
(iii) Structural diffusion, as a direct outcome of hole generation and coarse graining (mixing ?) is giving rise to a transient approach to a new collisionless equilibrium, before classical diffusion processes do their final work.

One may, therefore, conclude that this scenario, being valid in the LINEARLY STABLE regime, expresses a NEW PARADIGM OF PLASMA STABILITY.

As a footnote we mention that a fusion plasma, which is typically driven by many sorts of currents, e.g. due to ohmic heating, neutral beam injection, wave heating, profile control, etc., has to overcome this non-linear growth of trapping structures and the associated onset of anomalous, structural diffusion during which radial losses are expected to be strongly enhanced, even if experimentally all linear instabilities can be suppressed.

Theoretically, a number of open questions remain. One of them is concerned with the underlying mechanism of growth of holes. Although we have learned that holes of zero and/or negative energy are preferentially excited, we still do not know the underlying mechanism(s), as an energy principle, in which trapping effects are incorporated, is still absent. Also, the simultaneous excitation of a pair of holes remains unclear from an analytic point of view, and so on. Finally, we mention that the fast reconnection processes, thought to be responsible for coronal mass ejection (CME) processes taking place in the solar corona (S. T. Wu, 2005, private communication), may be triggered by the anomalous, structural diffusion (enhanced resistivity), as no linear electrostatic two-stream instability seems to exist in this part of the corona.

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