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Kinetic theory of periodic hole and double layer equilibria in pair plasmas

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Abstract. The existence of manifestly nonlinear electrostatic modes in pair plasmas is shown analytically by means of the quasi-potential method applied to the Vlasov–Poisson system. These modes owe their existence to the trapping of particles in the potential trough(s) and are typically characterized by a notch in the particle distribution functions at resonant velocity, forming vortices in phase space. Both entities, wave structure $\Phi(x)$ and phase velocity v_0, are uniquely characterized by two parameters, the periodicity parameter k_0 and the spectral parameter B. Whereas $k_0 = 0$ describes double layers, with a phase velocity in the thermal range, $k_0 \neq 0$ represents a periodic wave train which can propagate with two rather distinct phase velocities. One is related to the fast plasma wave, the other one to the slow acoustic mode.

Contents

1. Introduction

In recent years, pair plasmas consisting of two components with equal mass and opposite charge have received considerable attention from (astro)physicists. One reason is that electron–positron plasmas belonging to this category have been created in laboratories [1]–[3] and are thought to be generated naturally by pair production in high-energy processes occurring in many astrophysical environments such as the early universe, neutron stars, active galactic nuclei or pulsar magnetospheres [4]–[7].

Another reason is that plasmas of this type have unique thermodynamic properties. Namely, mass equality results in a high momentum transfer during binary collisions such that both species relax on the same timescale to thermodynamic equilibrium and hence acquire more or less the same temperature. This also implies that both species evolve more or less equally in dynamical processes. In contrast to an ordinary electron–ion plasma with different masses, there is no regime in which the dynamical evolution is dominated by one species. One well-known consequence is, for example, that whistler waves are missing [8].

A further argument of special interest is the considerable simplification in the mathematical description of such plasmas with the possibility of making progress in areas that could not be clarified so far, for example, the influence of nonlinearity on the evolution of a driven plasma.

A drawback of an electron–positron pair plasma, on the other hand, is that pair annihilation and radiation cooling by cyclotron emission gives rise to a rather short life-time, generally too short to allow the experimental investigation of collective plasma effects. Recently, however, another pair plasma—a fullerene pair plasma—was able to be produced without these obstacles [9].

In the present paper, a kinetic model for the simplest case of wave-like collective excitations, namely for electrostatic modes for which self-consistent particle trapping represents a non-ignorable ingredient, is presented.

2. Basic equations

In the following, reference is made to the pseudo-potential method for solving the corresponding Vlasov–Poisson system as introduced in [10] and further developed in a series of papers [11]–[15]. Denoting positive (negative) ion quantities by a $+$ $(-)$ index, assuming equal temperatures, $T_+ = T_- \equiv T$ and equal masses, $m_+ = m_- \equiv m$, we get the following 1D Vlasov–Poisson system for a pair plasma:

$$[\partial_t + v\partial_x \pm \Phi'(x, t)\partial_v]f_{\mp}(x, v, t) = 0, \tag{1a}$$

$$\Phi''(x, t) = \int dv\, f_- - \int dv\, f_+ \equiv n_- - n_+, \tag{1b}$$

where space (x), time (t) and electric potential (Φ) are normalized in units of the Debye length λ_D, inverse plasma frequency ω_p^{-1} and T/e of the unperturbed state. The velocity is, therefore, normalized by the thermal velocity $\sqrt{T/m}$. Looking for a time-independent travelling wave solution, we can perform a Galilean transformation, $x - v_0 t \to x$, $v - v_0 \to v$, where v_0 is the phase velocity, to get rid of the time derivative in (1a). Making use of the second procedure of [11] for which $0 \leqslant \Phi(x) \leqslant \Psi$ holds, where Ψ is the amplitude, these two simplified equations

are then solved by

$$f_{\mp}(v, \Phi) = \frac{N_{\mp}}{\sqrt{2\pi}} \left\{ \theta(\epsilon_{\mp}) \exp\left[-\frac{1}{2}\left(\sigma\sqrt{2\epsilon_{\mp}} + v_0 \right)^2 \right] + \theta(-\epsilon_{\mp}) \exp\left(-\frac{v_0^2}{2} \right) \exp\left(-\beta\epsilon_{\mp} \right) \right\}, \quad (2)$$

where the normalization constants are given by $N_- := 1 + k_0^2\Psi/2$ and $N_+ := 1 + A$. The single particle energies, which are constants of motion, are given by $\epsilon_{\mp} := \frac{v^2}{2} - \Phi_{\mp}$, where we define $\Phi_- := \Phi$ and $\Phi_+ := \Psi - \Phi$ resp. The separatrix in the phase space of both species is then given by $\epsilon_{\mp} = 0$, separating free ($\epsilon_{\mp} > 0$) from trapped ($\epsilon_{\mp} < 0$) ions.

At the position where trapped ions are absent (i.e. $\Phi = 0$ for negative and $\Phi = \Psi$ for positive ions), the distributions reduce to a shifted Maxwellian, $f_{\mp} \sim \exp\left[-\frac{1}{2}(v + v_0)^2 \right]$. In (2), $\theta(x)$ represents the Heaviside step function, σ is the sign of v, β is the trapping parameter, controlling the state of trapped ions, A is a normalization constant and k_0 is a parameter related to the actual wavenumber k of the periodic wave, as will be discussed later. Note further that both distributions are continuous across the separatrix and that identical trapping conditions are assumed for both species.

A velocity integration [10]–[15] yields the densities, which in the small amplitude limit, $\Psi \ll 1$, become

$$n_{\mp}(\Phi) = N_{\mp} \left[1 - \frac{1}{2} Z_r'\left(\frac{v_0}{\sqrt{2}} \right) \Phi_{\mp} - \frac{4}{3} b(\beta, v_0) \Phi_{\mp}^{3/2} + \cdots \right]. \quad (3)$$

We fix A in N_+ by demanding that at $\Phi = 0$, where trapped negative ions are absent, the density of the positive ions is unity, and get $A = \frac{1}{2} Z_r'(v_0/\sqrt{2})\Psi + \frac{4}{3} b(\beta, v_0)\Psi^{3/2}$. The function $b(\beta, v_0)$, representing the trapping effect, is defined by

$$b(\beta, v_0) = \frac{1}{\sqrt{\pi}} \left(1 - \beta - v_0^2 \right) \exp\left(-\frac{v_0^2}{2} \right), \quad (4)$$

and $Z_r(x)$ stands for the real part of the plasma dispersion function, which is defined by $Z_r(x) = \frac{1}{\sqrt{\pi}} P \int ds \frac{\exp(-s^2)}{s-x}$, where P means the principal value.

Inserting (3) into the rhs of Poisson's equation (1b), we can formally write it as a classical equation of motion $\Phi'' = -V'(\Phi)$ and get by an integration $\Phi'(x)^2/2 + V(\Phi) = 0$, where $V(\Phi)$, using $V(0) = 0$ and the explicit expression for A (resp. N_+), is given by

$$-V(\Phi) = \frac{k_0^2\Psi\Phi}{2} - \frac{1}{2} Z_r'\left(\frac{v_0}{\sqrt{2}} \right) \Phi^2 - \frac{8}{15} b(\beta, v_0) \left[\Phi^{5/2} + (\Psi - \Phi)^{5/2} + \frac{5}{2}\Psi^{3/2}\Phi - \Psi^{5/2} \right]. \quad (5)$$

From the pseudo-potential $V(\Phi)$ given by (5), we learn that two conditions must be satisfied to get a solution:

$$\text{(i) } V(\Psi) = 0; \qquad \text{(ii) } V(\Phi) < 0, \quad \text{for } 0 < \Phi < \Psi. \quad (6)$$

The first condition becomes

$$-\frac{1}{2} Z_r'\left(\frac{v_0}{\sqrt{2}} \right) = -\frac{k_0^2}{2} + B \equiv D, \quad (7)$$
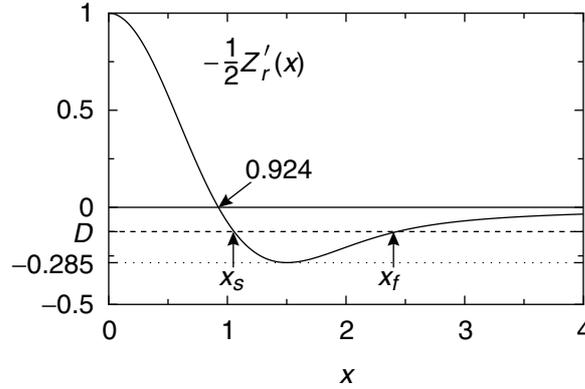
Figure 1. The function $-\frac{1}{2}Z_r'(x)$ and two solutions of the NDR, given by (7) in the text, for $-0.285 < D < 0$.

which represents the nonlinear dispersion relation (NDR) because it is the determining equation for the phase velocity v_0 depending on k_0, β and Ψ. In (7), B is given by

$$B = \frac{4}{3}b(\beta, v_0)\sqrt{\Psi}. \tag{8}$$

By use of (7), the pseudo-potential (5) simplifies to

$$-V(\Phi) = \frac{k_0^2}{2}\Phi(\Psi - \Phi) - B\left\{\Phi(\Psi - \Phi) + \frac{2}{5\sqrt{\Psi}}\left[(\Psi - \Phi)^{5/2} - (\Psi^{5/2} - \Phi^{5/2})\right]\right\}. \tag{9}$$

The NDR (7) and the pseudo-potential $V(\Phi)$ (9) uniquely characterize the BGK-like (Bernstein–Greene–Kruskal [16]) wave structure. They agree with (24) and (25) of [11] if the limit of equal temperatures, masses and trapping parameters is taken there and a drift between the negative and positive species is neglected.

From (9) we immediately see that $V(\Phi)$ is symmetric around $\Phi = \Psi/2$, which implies that we have either to deal with periodic wave solutions $\Phi(x)$ (if $-V'(0) = V'(\Psi) > 0$) or with double layer solutions (if $-V'(0) = V'(\Psi) = 0$, $V''(0) = V''(\Psi) < 0$). A double layer solution is a potential which connects monotonically two asymptotic states $\Phi = 0$ and $\Phi = \Psi$.

3. Harmonic waves

To analyse the solution further, we first select a trapping condition for which B in (8) vanishes. The pseudo-potential (9) then becomes $V(\Phi) = \frac{k_0^2}{2}\Phi(\Psi - \Phi)$, which results in $\Phi(x) = \frac{\Psi}{2}[1 + \cos(kx)]$. This is the harmonic wave limit in which $k = k_0$, which means k_0 directly represents the wave number. The phase velocity $v_0 =: \omega/k$ of this wave follows from (7) with $B = 0$ and $k_0 = k$. Figure 1 illustrates the possible solutions. The value D, given by $-k^2/2$, is negative and one has two solutions, $x_s \equiv \omega_s/\sqrt{2}k$ and $x_f \equiv \omega_f/\sqrt{2}k$, provided that $-0.285 \leqslant -k^2/2 \leqslant 0$ or $0 \leqslant k \leqslant 0.755$. The corresponding NDR ω versus k is shown in figure 2. We see that there is a fast and a slow mode which join at $k = 0.755$, $\omega = 1.133$ which is termed the transition point. For small k we can use the Taylor expansions of $-\frac{1}{2}Z_r'(x)$ at $x_0 = 0.924$ and
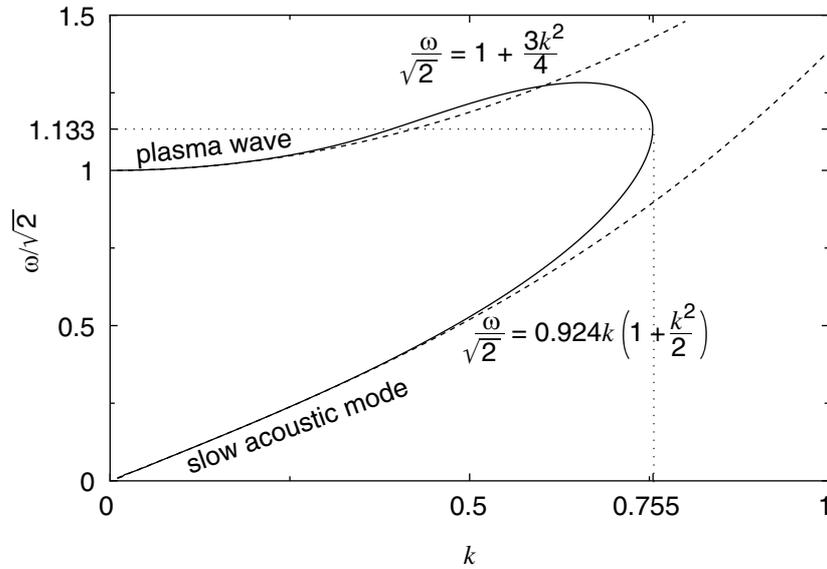
Figure 2. The NDR for a harmonic wave structure with $\omega := k v_0$ (solid line). The long-wavelength limit, $k \ll 1$, is drawn by dashed lines showing the fast plasma wave and the slow acoustic mode.

at infinity [10]–[15] to get

$$\frac{\omega_f}{\sqrt{2}} = 1 + \frac{3}{4} k^2; \qquad \frac{\omega_s}{\sqrt{2}} = 0.924 k \left(1 + \frac{k^2}{2} \right). \tag{10}$$

The fast mode in the long-wavelength limit is, hence, nothing else but an ordinary plasma wave, where the denominator $\sqrt{2}$ reflects the fact that both species contribute to the oscillatory motion in the same manner doubling the density in the plasma frequency expression.

The slow mode for small k on the other hand has nothing to do with an ordinary ion acoustic wave as sometimes suggested. One can easily see that ion acoustic waves, such as whistler waves, are absent in a plasma pair. The reason is that the linear Landau dispersion relation $k^2 - Z'(\omega/\sqrt{2}k) = 0$ has weakly damped solutions only for large ω_f/k corresponding to the fast plasma wave.

This slow mode is a real nonlinear mode and is termed the slow acoustic mode [13, 17]. In an ordinary electron–ion plasma, the corresponding slow ion (electron) acoustic mode is the mode on which ion (electron) holes rest propagating near ion (electron) thermal velocity [12, 13]. To distinguish them from the ordinary ion acoustic wave (resp. fast Gould–Trivelpiece electron acoustic mode in a cylindrical plasma), the term 'slow' was added, a nomenclature we adopt also for the present case.

Although the corresponding NDR, (7) with $B = 0$, looks rather 'linear', this mode only exists as a true solution of the full nonlinear Vlasov equation, keeping in mind that $B = 0$ has to be satisfied (see below). Formally, it looks like a van Kampen mode (with a vanishing δ-function contribution to the perturbed distribution function [11]) but the latter satisfies the truncated linearized Vlasov equation only. We prefer a solution of the full nonlinear Vlasov equation, and can accept the slow acoustic mode only, for which $\partial_v f_1$ is of the same order as $\partial_v f_0$,

where f_0 (f_1) is the unperturbed (perturbed) distribution function, and hence not negligible. A linearization procedure, hence, would fail to describe the slow acoustic mode.

Whereas for the fast, long-wavelength plasma wave, the condition $B = 0$ can be satisfied by the exponential factor in (4), and hence the wave behaves approximately like a fluid-like linear wave, for the slow acoustic, long-wavelength mode with $v_0 \approx 1.307 \equiv 0.924\sqrt{2}$, the factor $(1 - \beta - v_0^2)$ has to vanish in (4) which implies $\beta = -0.708$. Only a notch in the trapped ion distribution provides the proper existence condition for the slow acoustic mode. In this region, $|\partial_v f_1| \ll |\partial_v f_0|$ is definitely violated and hence this mode exists only as a nonlinear mode.

4. Cnoidal waves

Next we allow for nonzero values of B, $B \neq 0$, to get generalized periodic wave solutions, described by Jacobian elliptic functions, called Cnoidal or Snoidal waves [10, 12, 15]. This follows from the quadrature of the integrated Poisson's equation with the quasi-potential $V(\Phi)$ given by (7) for which the non-harmonic second term now contributes. This generally gives rise to a specific, slowly decaying spectral decomposition of $\Phi(x)$ for $|B| \ll 1$ but all the more for finite B, $-0.285 < B \lesssim \mathcal{O}(1)$. We will call B, which incorporates the effects of trapping, wave amplitude and phase speed the spectral parameter, because it controls the spectral content of the wave structure.

The NDR for a given $B \neq 0$ follows from (7) yielding $v_0 = \frac{\omega}{k} = \frac{\omega}{k_0}\ell(k_0, B)$, where $\ell(k_0, B)$ reflects the relationship between the periodicity parameter k_0 and the actual wavenumber k, which follows from the expression we obtained for $\Phi(x)$ and which depends on B. If $B = 0$, we have $\ell(k_0, 0) = 1$ and $k_0 = k$, as used already for the harmonic wave. For Cnoidal waves, $\ell(k_0, B)$ generally differs from unity, but can be found [10, 12, 15]. Hence, by plotting the quasi-NDR $\omega_0(k_0) := k_0 v_0$ as a function of k_0, as done in figure 3, we can infer on the correct NDR via $\omega(k) = k v_0 = \frac{\omega_0(k_0)}{\ell(k_0, B)}|_{k_0=k_0(k, B)}$, where $k_0(k, B)$ is the inverse of $k(k_0, B) \equiv k_0/\ell(k_0, B)$ for B fixed. Assuming $k(k_0, B)$ is monotonic in k_0 (for which good reasons exist), the quasi-NDR $\omega_0(k_0)$ then informs us qualitatively about the correct behaviour of the phase velocity v_0 in terms of k.

The quasi-NDR, as exhibited in figure 3, shows that a change of B has a rather strong influence on the dispersion properties, especially for small k_0 and B around zero. If $k_0 \to 0$ and $-B > 0$, equation (7) tells us that the fast mode has the solution $v_0 = \omega_0(k_0)/k_0 \equiv (-B)^{-1/2} \gg 1$, which means that $\omega_0(k_0) = (-B)^{-1/2}k_0$ tends to zero as $k_0 \to 0$. On the other hand, if $B > 0$, a fast mode solution only exists for $k_0 > \sqrt{2B}$. Whereas a continuous change of B gives rise to continuous dependence of the slow acoustic branch on B, the fast branch solution behaves in a discontinuous way: becoming acoustic-like near $k_0 = 0$, for $0 < -B$; and non-existent for $k_0 < \sqrt{2B}$, $B > 0$. The plasma wave solution of figure 2, for which $B \equiv 0$, is approached rather differently from both sides, $B \neq 0$, the difference being the stronger the smaller k_0 is. Hence, any deviation from $B = 0$ yields a rather drastic change in the dispersion characteristics.

The turning point, where both solutions join, is again given by $-k_0^2/2 + B = -0.285$, which is the minimum of the lhs of (7) with the solution $v_0/\sqrt{2} = 1.5$ or $\omega_0/\sqrt{2} = 1.5k_0$, shown by the dotted line in figure 3. If B tends to -0.285, both solutions of (7) are concentrated around $v_0/\sqrt{2} = 1.5$ giving a shrinking area of existence, as seen e.g. for $B = -0.25$.

The (quasi-)dispersion characteristics of the trapped particle modes are, hence, rather complex and involved and by no means predictable by any linear wave theory.
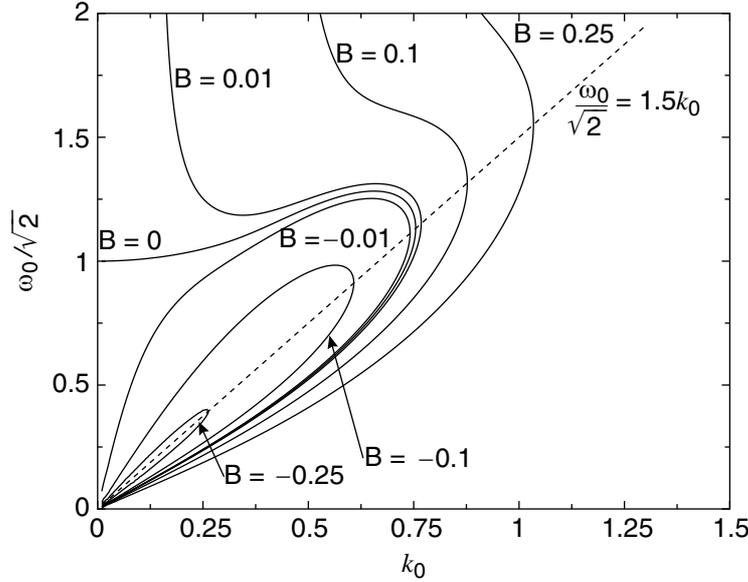
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Figure 3. The quasi-NDR, $\omega_0 := k_0 v_0$ as a function of k_0, for several values of the spectral parameter B with $-0.285 < B$. The fast and the slow mode join at $\omega_0/\sqrt{2} = 1.5k_0$ (dotted line). For $B < 0$ both modes become acoustic-like in the small k_0 limit, whereas for $B > 0$ no fast mode exists for $k_0 < \sqrt{2B}$.

For a given solution v_0 of the NDR and a given wave amplitude Ψ, the trapping condition (8), with $b(\beta, v_0)$ given by (4), becomes

$$- \beta = \frac{3}{4}\sqrt{\frac{\pi}{\Psi}} \exp\left(\frac{v_0^2}{2}\right) B + v_0^2 - 1, \tag{11}$$

showing that the depression of the distribution function in the trapped region increases with B.

5. Double layers and general existence diagram

Finally, let us discuss the case of a vanishing k_0, $k_0 = 0$. In this case, the pseudo-potential $V(\Phi)$ simplifies with $\phi := \Phi/\Psi$ to

$$- V(\phi) = B\Psi^2 \frac{2}{5}\left[(1 - \phi^{5/2}) - (1 - \phi)^{5/2} - \frac{5}{2}\phi(1 - \phi)\right]. \tag{12}$$

From this expression we can see that $V(0) = V'(0) = 0$ and $V''(0) = -B/2$, which holds also for the argument $\phi = 1$. We, hence, arrive at a double layer (DL) solution, provided that the second condition (ii) in (6) is satisfied, becoming $B > 0$. With $B > 0$ (and $k_0 = 0$), the NDR (7) together with figure 1 and $D = B > 0$ tell us that a DL solution only exists on the slow acoustic branch with $0 \leqslant v_0/\sqrt{2} < 0.924$ corresponding to $1 \geqslant B > 0$. In view of (11) this implies that a standing DL ($v_0 = 0$) requires a notch ($\beta < 0$) in the trapped region of the distribution function(s) if $B > \frac{4}{3}\sqrt{\frac{\Psi}{\pi}}$, which becomes deeper as B increases.
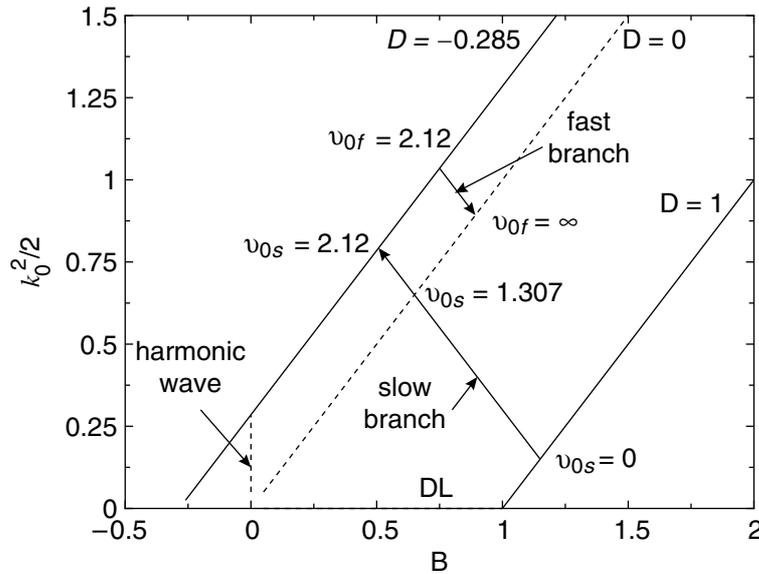
Figure 4. The existence diagram of nonlinear wave solutions in the $(B, k_0^2/2)$-parameter space. Waves with constant phase velocities lie on a straight line $D = \text{const}$. No solution exists for $B \leqslant 0$ and small $k_0^2/2$.

From the density expressions (3), $k_0 = 0$ and use of the NDR, we learn that $n_\mp(\Phi) = 1 + B\Phi_\mp[1 - \sqrt{\frac{\Phi_\mp}{\Psi}}]$, where $\Phi_\mp$ have been defined earlier. Both are hump-like approaching unity as $\Phi \to 0$ and $\Phi \to \Psi$, the positive jump being deformed such that $n_- \gtrless n_+$ if $\Phi \lessgtr \Psi/2$.

There is no density jump across the DL in contrast to what is known from fluid-like dissipative shock structures.

Finally, in figure 4 we sketch the region of existence of solutions in the $(k_0^2/2, B)$-parameter space, being essentially restricted to the strip $-0.285 \leqslant D \leqslant 1$. A line $D = \text{const}$ corresponds to a fixed phase velocity. Whereas in the strip $1 \geqslant D > 0$, only the slow branch exists, corresponding to $0 \leqslant v_{0s} < 1.307$; there are two solutions at every point of the strip $0 > D \geqslant -0.285$, corresponding to a slow mode with $1.307 < v_{0s} \leqslant 2.12$ and a fast mode with $2.12 \leqslant v_{0f} < \infty$. Double layer solutions are found for $k_0 = 0$ and $0 < B \leqslant 1$, and harmonic waves for $B = 0$ and $0 < k_0^2/2 \leqslant 0.285$. No solution exists for $B \leqslant 0$ and sufficiently small k_0.

How to identify a mode? From the measured dispersion characteristics one can derive the background density and the plasma frequency (from the plasma wave branch) and the temperature (from the slow acoustic branch) and hence the normalized values of ω and k. Then one approximates ω by ω_0 and k by k_0 and gets the value of B from the quasi-NDR, namely figure 3. Using (11) and the value of Ψ, which follows from the strength of the associated density fluctuations, one obtains the value of β and hence the information about the status of trapped particles. Finally, to test and to improve the approximation, one may calculate $k(k_0, B)$ and compare it with k_0 and use this k instead of k_0 as a next iteration.

6. Concluding remarks

A pair plasma, due to its reduced parameter dependence and associated symmetry, provides an optimum laboratory under which further progress in the understanding of the complexity of

collective phenomena including anomalous transport and turbulence in driven collisionless or weakly collisional plasmas [15] can be made. In this respect, of high theoretical interest is the question of whether solitary hole or hump solutions with negative energy can exist [18, 19], which may nonlinearly destabilize a current-carrying plasma even below the threshold of linear two-stream instability. An answer can be found e.g. by releasing the constraint of equal trapping, on which this paper relies, and by looking at other types of waves.

Acknowledgments

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