

On the (de)focusing nature of wall impedances for hadron beams in circular accelerators and storage rings

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On the (de)focusing nature of wall impedances for hadron beams in circular accelerators and storage rings

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Abstract. The hole theory (Schamel H 1997 *Phys. Rev. Lett.* **79** 2811) of longitudinal solitary structures superimposed on coasting beams, as has been seen recently in experiments, is evaluated in the zero-resistance limit to get new insight into the (de)focusing properties of wall impedances. It is found that the standard picture on (de)focusing, according to which a space-charge-dominated impedance is defocusing if the beam is below transition energy, as an example, does not always hold. Namely, this property (and picture), which can already be obtained by a simple waterbag model, remains true only when $\mathcal{L} < \mathcal{L}_c$, where \mathcal{L} is the wall inductance per unit length and \mathcal{L}_c is given by $\mathcal{L}_c = \mu_0/4\pi\beta^2\gamma^2$. For $\mathcal{L} > \mathcal{L}_c$, however, just the opposite (de)focusing property holds. Responsible for this new feature for coasting beams are wake-fields which are taken into account in a next-order self-consistent perturbation theory. It is argued, furthermore, why for bunched beams a similar result might also be expected.

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1. Introduction

The highly collisionless nature of beams in particle accelerators and storage rings sometimes causes problems for the achievement of higher and higher beam currents and charge densities and hence to reach one of the main goals in the commissioning and operation of modern high-energy machines. One problem e.g. arises from the formation of coherent structures which are superimposed on the designed beam, as observed recently in coasting [1, 2] as well as in bunched [3]–[5] beams. In both cases, a theoretical explanation [5]–[11] has been given by the Vlasov–Poisson description of the longitudinal beam dynamics, more precisely by vortex-type equilibria in the longitudinal phase space, reminiscent of the holes (and humps) found in plasmas [12]. In fact, in both physical domains, kinetic holes and humps turn out to be omnipresent and easily excitable structures.

Whereas a hump-like pattern appears to be advantageous in increasing the local phase-space density and, hence, the beam quality, holes are less favourable as they have the opposite effect. A standard remedy to remain in the favourable regime is to select the wall impedance and to control the longitudinal dynamics in such a way that an emerging localized perturbation is preferentially of hump-type, a case being usually referred to as a focusing impedance.

In a recent paper [5], in which the solitary hole (resp. hump) concept for coasting beams could also be transferred to bunched beams, yielding a consistent theoretical description of some observations made on RHIC at Brookhaven, the question arose as to whether a simplified model can already explain the main (de)focusing property of impedances or whether the full theoretical description of solitary waves is demanded.

This simplified model, which is probably due to Sacherer [13], predicts that below transition, a space-charge-dominated impedance is defocusing, leading to a hole in the beam, whereas it is focusing, leading to a hump, if the beam is above transition. For an inductive-dominated impedance the opposite property holds.

Obviously, in this classification, only the sign of the purely reactive impedance enters, which determines the (de)focusing nature of the beam.

In the present paper, however, we shall show that the situation is more complex and delicate than previously thought. We point out the drawback and limitation of Sacherer's model and offer a way to pose and solve the problem adequately. After a short introduction into the governing equations valid for coasting beams in the purely reactive impedance limit, we first rederive the standard picture of (de)focusing impedances for piecewise constant particle distributions (the so-called waterbag model in plasma physics [14]), show its limitations and then perform

the solitary wave analysis taking into account the next-order effect in an expansion scheme with $\gamma^{-2} \ll 1$ being the expansion parameter, where γ is the relativistic factor.

2. Basic equations

The analysis is based on the kinetic description of the longitudinal beam dynamics given by the Vlasov–Poisson system [9]

$$[\partial_t + p\partial_z - \epsilon\partial_p]f = 0, \quad (1)$$

$$\epsilon'' = \bar{\mu}\epsilon + \bar{\alpha}\lambda_1', \quad (2)$$

where λ_1 is the perturbed line density. In (1) and (2), the normalization of [6, 9] has been adopted, which implies for the line density $\lambda = \int dp f$:

$$\int_{-1/2}^{+1/2} dz \lambda(z) = 1. \quad (3)$$

In the second equation the limit of a purely reactive impedance has already been taken. This system assumes a highly relativistic hadron beam with $\gamma \gg 1$, an excited field structure according to a transverse magnetic (TM) mode and radially constant beam density. In the derivation of (2), which can be found in the appendix of [9] (see also [6]) the elimination of the transversal field components by means of Ampère's and Faraday's law yield an integrodifferential equation for the longitudinal electric field, (A6) of [9], which can be solved iteratively up to $O(1/\gamma^2)$. In this next order, a field correction appears which turns out to be electromagnetic in nature, reminiscent of a wake field. Then, by relating the leading part of the longitudinal electric field with the wall electric field, expressing the latter linearly by the wall inductance, one arrives at (2) (resp. (11) of [9]).

The various dimensionless quantities in (1) and (2) are defined as in [9]. This means that the constants $\bar{\mu}$ and $\bar{\alpha}$ in (2) are given by

$$\bar{\mu} = \frac{\mu}{1-L}, \quad \bar{\alpha} = \alpha \frac{g_0 - L}{1-L}, \quad (4)$$

where L is the dimensionless wall inductance and g_0 represents the capacitive space-charge impedance of the beam. The parameters μ and α , where μ is proportional to $(\gamma R_0/b)^2$ and α proportional to ηN , are typically very large, so that one is tempted to solve (2) approximately by letting both terms on the right-hand side balance each other. Note that R_0 is the large radius of the design trajectory, b the radius of the circularly shaped cross-section of the toroidal vessel, N the total number of particles in the beam and η is the slip factor.

3. Quasineutrality, Sacherer's model and its limitations

This procedure, which corresponds to the assumption of quasineutrality in the treatment of plasmas, and which for brevity will be termed the quasineutrality assumption (QN) further on, leads to

$$\Phi = \frac{\bar{\alpha}}{\bar{\mu}} \lambda_1 \equiv k \lambda_1, \quad (5)$$

where we have introduced the potential Φ via $\epsilon = -\Phi'$ and already have performed one integration of (2). This simple proportionality between Φ and the perturbed line density λ_1 is well known for purely reactive impedances as found e.g. in [15]. The proportionality constant k is given by (4) and becomes

$$k = \frac{\alpha}{\mu}(g_0 - L). \quad (6)$$

Hence, the single-particle Hamiltonian, determining the trajectories (characteristics) of the particles in the Vlasov equation (1), can be written as

$$H(\Phi, p) := \frac{p^2}{2} - \Phi = \frac{p^2}{2} - k\lambda_1 \equiv \frac{p^2}{2} - k \int f_1 dp, \quad (7)$$

where f_1 is the perturbed distribution function in the presence of Φ . If Φ is a positive, localized perturbation of bell shape, the phase-space pattern of trajectories shows an O-type separatrix, given by $H = 0$, sometimes also called a cateye phase pattern, which separates free from trapped particles. Note the similarity to electron holes known in plasma physics.

Now, Sacherer's simple model to characterize Φ and f_1 is to assume that both are piecewise-constant, i.e. Φ and f_1 are nonzero in $|z| < l$ and zero outside (see also figure 1 in [5]). This already suffices to determine the focusing nature of the complex impedances. Namely, using the constancy of H along the separatrix, we get using (7)

$$0 = H(0, 0) = \frac{p_s^2}{2} - \Psi = \frac{p_s^2}{2} - k \int_{-p_s}^{+p_s} dp f_1 = \frac{p_s^2}{2} - 2kp_s f_1, \quad (8)$$

where Ψ is the maximum value (amplitude) of Φ in $|z| < l$ and p_s is the momentum of a particle with zero energy. We hence get $p_s = 4kf_1$ which has to be positive and we arrive at $kf_1 > 0$, which by insertion of (6) becomes

$$\alpha(g_0 - L)f_1 > 0, \quad (9)$$

noting that μ is strictly positive. We, therefore, immediately get that f_1 is positive, corresponding to a bunching of the beam or to the focusing nature of the beam, provided that $\alpha(g_0 - L)$ is positive. Since α carries the sign of slip factor this implies that above transition ($\alpha > 0$) an impedance in which space-charge dominates ($g_0 > L$) is of focusing nature. Below transition, on the other hand, this same impedance will be defocusing since f_1 is negative, corresponding to a hole in the beam distribution.

This situation, resulting from (9), is illustrated in figure 1 which shows schematically the (de)focusing nature of a beam in the case of a space-charge-dominated reactive impedance ($g_0 > L$) and $L < 1$. Plotted are the actual dimensionless electrostatic potential, for which $\Phi \sim \eta q \tilde{\Phi}$ holds, and the corresponding phase-space pattern, in which the dark area represents a region of higher phase-space density. Focusing (defocusing) occurs above (below) transition energy $\eta > 0$ ($\eta < 0$) commonly denoted as a negative (positive) mass system.

Switching from an antiproton ($q < 0$) to a proton ($q > 0$) beam keeping the other parameters fixed, one merely changes the polarity (sign) of $\tilde{\Phi}$ without affecting the (de)focusing nature of the beam. Hence, the right column corresponds to focusing being valid as long as $L < 1 < g_0$.

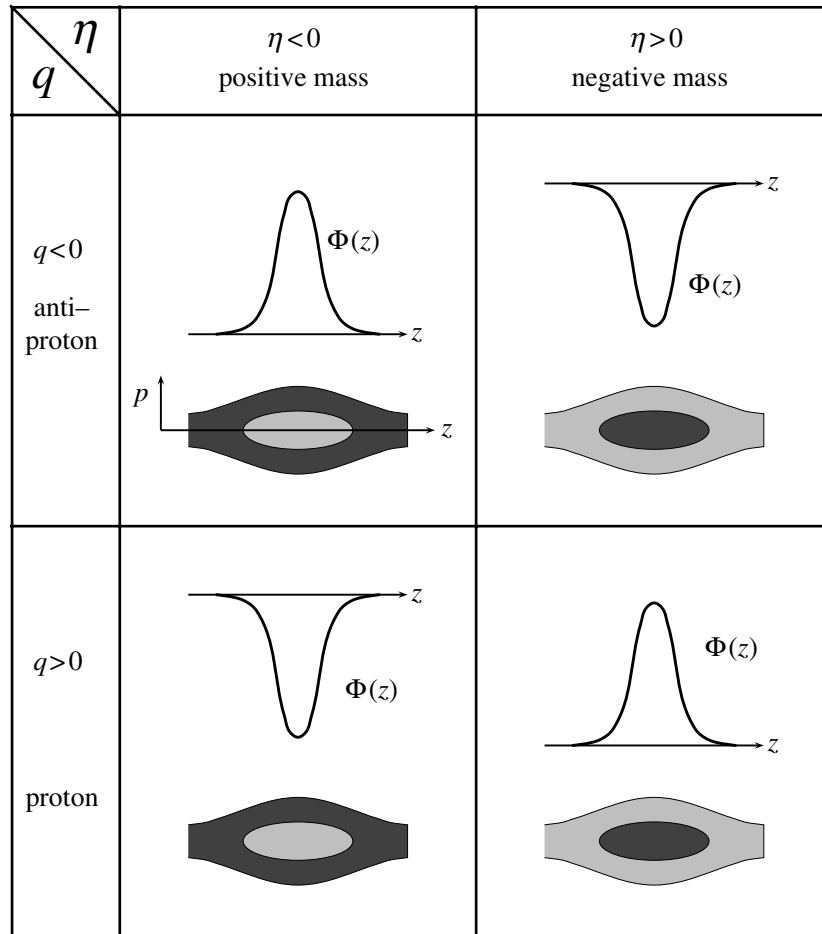


Figure 1. Standard (de)focusing properties of impedances, valid for $L < 1 < g_0$. When $1 < L$ the chart is diagonally inverted, which means, for example, that for $\eta < 0$ (positive mass) the system is focusing, while for $\eta > 0$ (negative mass) the system is defocusing.

This focusing property of impedances, denoted further on as standard, seems to be generally accepted in the beam community, and Sacherer's model seems to provide a very simple explanation. (Note that 'self-consistency' comes out automatically, since Ψ is directly determined by k , p_s and f_1 .)

However, at least three objections have to be raised which mask this 'derivation' and ask for a revision and a more appropriate definition of the (de)focusing nature of impedances.

1. Typical physical quantities are continuous rather than discontinuous, asking, for example, for more smooth distributions (rather than waterbag distributions).
2. A localized smooth potential Φ , which is typically bell-shaped with $0 \leq \Phi \leq \Psi$, has a negative curvature, $\Phi'' < 0$, at its centre, where $\Phi = \Psi$ (rather than equipped with a vanishing curvature).
3. The width l of a solitary potential is usually determined by its amplitude and status of trapped particles (rather than kept unspecified and hence left arbitrary).

Or, in other words, QN can only be assumed in the region far away from the structure, where the beam is unperturbed, and not at its centre. Therefore, a new assessment of the (de)focusing conditions is called for, which removes these objections.

The derivation, presented next, closely follows that of [9].

4. Solitary wave solutions and the criterion for (de)focusing

We are looking for a small-amplitude solitary wave, $0 \leq \Phi \leq \Psi \ll 1$, which propagates as a stationary structure with the velocity of the design trajectory along the torus, i.e. its phase velocity is assumed to be zero in the co-rotating frame which is, hence, the wave frame. Making use of the potential method [6]–[12], we first solve the time-independent Vlasov equation by the ansatz [11]

$$f(z, p) = \frac{1 + K}{\sqrt{2\pi}} \{ \exp(-H)\theta(H) + \exp(-\beta H)\theta(-H) \}, \quad (10)$$

where H is the single-particle energy (7), being a constant of motion, and θ is the Heaviside step function. The first term refers to the free or untrapped particles, the second one to the particles which are trapped in the potential. The distribution (10) is continuous everywhere, inclusively at the separatrix $H = 0$. A negative value of β corresponds to a notch in the trapped particle region in phase space given by $|p| < p_s = \sqrt{2\Phi}$, i.e. the distribution is depressed in this region. If β is positive but small we have a more flat-topped distribution whereas for $\beta \geq 1$ a central hump arises in the distribution. It is this hump (notch) characteristic of the distribution which will be associated later with focusing (defocusing). Without loss of generality, we have assumed a Maxwellian unperturbed distribution $f_0(p)$ but a different, normalized $f_0(p)$ could also have been taken.

Integrating over the momentum we get the line density as a functional of Φ

$$\lambda(\Phi) = (1 + K) \left[1 + \Phi - \frac{4}{3} \bar{b}(\beta) \Phi^{3/2} + \dots \right], \quad (11)$$

where use of the smallness of Φ has already been made. In (11) $\bar{b}(\beta)$ is given by

$$\bar{b}(\beta) = \frac{1 - \beta}{\sqrt{\pi}}. \quad (12)$$

Using the normalization condition (3) and $\Psi \ll 1$ one can show that K is itself a small quantity, $K = \kappa\Psi$, where κ in the solitary wave limit is proportional to the pulse width l which has to be sufficiently small ($l \ll 1$). In this case the correction K in the normalization of (10) and (11) is negligible.

The perturbed line density then becomes

$$\lambda_1(\Phi) = \Phi - \frac{4}{3} \bar{b}(\beta) \Phi^{3/2} + \dots \quad (13)$$

Inserting this expression into the integrated version of (2) we get

$$\Phi'' = A\Phi + B\Phi^{3/2} \equiv -V'(\Phi), \quad (14)$$

where

$$A := \bar{\mu} - \bar{\alpha}, \quad (15a)$$

$$B := \frac{4}{3}\bar{\alpha}\bar{b}. \quad (15b)$$

The introduction of the ‘classical’ potential $V(\Phi)$

$$-V(\Phi) := \frac{1}{2}A\Phi^2 + \frac{2}{5}B\Phi^{5/2} \quad (16)$$

allows us to integrate (14) once to get the ‘classical’ energy

$$\frac{1}{2}\Phi^2(z) + V(\Phi) = 0. \quad (17)$$

An investigation of $V(\Phi)$ (for which $V(0) = 0$ was assumed) immediately shows that a bell-shaped potential $\Phi(z)$ only exists if two constraints are satisfied:

$$V(\Psi) = 0, \quad (18a)$$

$$V(\Psi) < 0, \quad \text{if } 0 < \Phi < \Psi. \quad (18b)$$

The first one yields

$$A + \frac{4}{5}B\sqrt{\Psi} = 0, \quad (19)$$

and relates A , B and Ψ . This relation is typically called a nonlinear dispersion relation (NDR) since it determines the phase speed of the structure in the co-rotating frame in terms of the other parameters. In our present investigation this speed was assumed to be zero (for nonvanishing phase velocities see [9]).

Making use of (19), we can rewrite $V(\Phi)$ from (16) and get

$$-V(\Phi) = -\frac{2}{5}B\Phi^2(\sqrt{\Psi} - \sqrt{\Phi}). \quad (20)$$

With this expression, (17) can be integrated to give

$$\Phi(z) = \Psi \operatorname{sech}^4(z/l), \quad (21)$$

which gives the desired bell-shaped profile of the potential.

The width l is found to be

$$l = \sqrt{\frac{20}{-B\sqrt{\Psi}}} \equiv \frac{4}{\sqrt{A}}, \quad (22)$$

where in the last step use of the NDR (19) was made.

This solution requires the second constraint (18b) to be fulfilled. Namely,

$$B < 0 \quad \text{or} \quad A > 0. \quad (23)$$

For a solitary perturbation to be sufficiently localized $l \ll O(1)$ holds. This implies via (22), (19) and an $\bar{\alpha}$ kept arbitrary, that $|\bar{b}|\sqrt{\Psi} \gtrsim O(1)$ or, because of (12), that $|\beta| \sim \Psi^{-1/2} \gg 1$. The beam is either in the focusing ($\beta \gg 1$) or in the defocusing ($-\beta \gg 1$) regime.

What remains to be investigated is under which beam and impedance conditions one or the other regime prevails.

It is easily seen from (23) that the sign of β follows from

$$\alpha \frac{g_0 - L}{1 - L} \beta < 0, \quad (24)$$

an inequality which consequently reflects the existence of solitary wave equilibria.

This is our new criterion for characterizing the (de)focusing nature of purely reactive impedances. It has to be confronted with the standard criterion (9), where f_1 plays the same role as β , i.e. both $\beta > 0$ and $f_1 > 0$ correspond to a hump. Now, an additional factor $1/(1 - L)$ appears in the inequality. This means that previous results about (de)focusing remain valid only as long as $L < 1$.

For $L > 1$, however, just the reverse results emerge. Accordingly, an inductance ($1 < g_0 < L$) is focusing ($\beta > 0$) rather than defocusing if the beam is above transition energy ($\alpha > 0$) and a space-charge ($1 < L < g_0$) is focusing if the beam is below transition ($\alpha < 0$).

If we plot a diagram, corresponding to figure 1, but now being valid for $1 < L < g_0$, we simply have to interchange in figure 1 the boxes diagonally, i.e. $A_{11} \leftrightarrow A_{22}$, $A_{12} \leftrightarrow A_{21}$, where A_{ik} is the box in the i th row and k -column. This implies that focusing is now found below transition energy.

We hence arrive at a complete reversal of the (de)focusing nature of wall impedances in the purely reactive limit in cases when $L > 1$ or, in dimensional form, referring to an inductance per unit length \mathcal{L} , when

$$\mathcal{L} > \mathcal{L}_c \equiv \frac{\mu_0}{4\pi\beta^2\gamma^2}. \quad (25)$$

For a relativistic beam ($\beta \approx 1$), \mathcal{L}_c is found to be

$$\mathcal{L}_c = \left(\frac{10}{\gamma}\right)^2 \quad (\text{nH m}^{-1}).$$

Owing to the γ^{-2} dependence this reversal of (de)focusing becomes effective earlier, the faster the beam.

We note that our evaluation is based on solitary, i.e. localized longitudinal perturbations, nonpropagating with the beam, a limitation which can easily be lifted by considering propagating solitary structures or structures exhibiting spatial periodicity, as was done in [11].

5. Solitary structures on bunched beams

Finally, we mention that a similar evaluation may also be done for bunched beams. In fact, restricting first the analysis to the case where wake fields are absent by using the Hamiltonian (4) of [5], we get the standard picture of (de)focusing for a bunched beam corresponding to Sacherer's model. Namely, a space-charge-dominated beam below (above) transition energy is focusing (defocusing) with $\ell > 0$ ($\ell < 0$), where ℓ is related to the net inductance [5]. Indeed, we find by using the iterative numerical procedure described in section 6 of [5], for $\ell > 0$ ($\ell < 0$) a

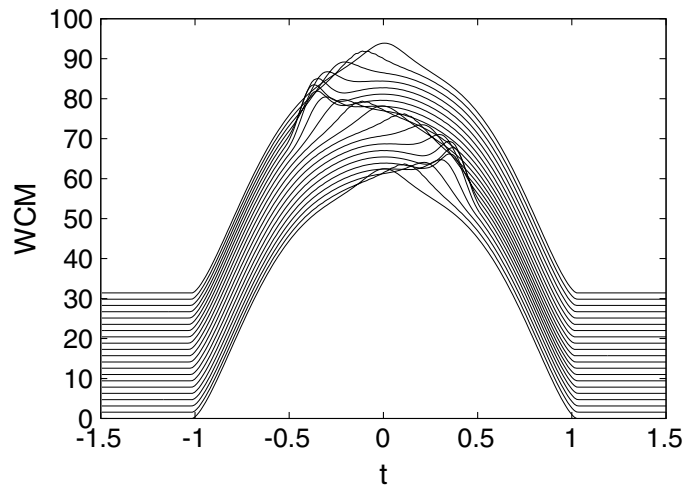


Figure 2. Mountain range plot of the wall current monitor (WCM) data that are measured from a bunched beam distribution function obtained with the iterative procedure described in [5], in which a positive trapping parameter $C_0 = 104213$ was used with $\ell = 0.01 > 0$. This gives rise to an additional hump in the WCM.

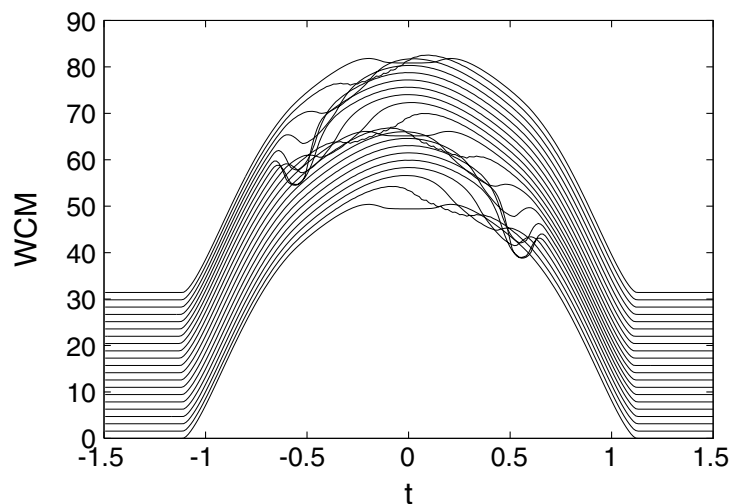


Figure 3. Mountain range plot or WCM (see figure 2) for a solution with $C_0 = -144208 < 0$, $\ell = -0.01 < 0$. A superimposed hole appears in the WCM.

hump (notch) in the distribution function which, by integration over the momentum, gives rise to a hump (notch) in the corresponding line density or wall current monitor data as seen in figure 2 (figure 3). Although stable holes have not yet been seen in bunched beams [5], they are possible within Sacherer's model. We may then speculate accordingly that a critical ℓ_c exists when wake fields are taken into account below which a reversal of (de)focusing properties holds. Whether this applies and explains the possible absence of holes in bunched beams should be addressed in future investigations.

6. Summary and conclusions

The present results have been derived under the assumptions of a transversally homogeneous beam of radius a and of a linear wall inductance representing the low-frequency response of a cavity ('broad-band impedance'). Under these conditions, the radial and longitudinal structure of the longitudinal electric field have been considered rigorously up to $O(\gamma^{-2})$. It would be of interest to learn whether and to what extent our results are modified if these limitations are lifted, e.g. by allowing a radial structure of the line density and a higher frequency cavity response model introduced by the boundary conditions. As a result, resonant and cut-off frequencies will be involved [16, 17], which may affect the (de)focusing properties of long and short bunches, worth studying further, qualitatively and quantitatively.

In summary, we have first shown how the standard (de)focusing model of Sacherer can be obtained from the general set of equations (1) and (2) by introducing the simplifications of quasineutrality and of waterbag distributions. Then, by releasing these constraints and applying the theory of solitary wave structures, we have derived an improved criterion (24) that reverses the standard (de)focusing picture of purely reactive impedances for coasting beams provided that (25) applies for the wall inductance per unit length. For bunched beams, a similar reversal may be expected, if wake field effects are taken into account.

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