



## Quantum corrected electron holes

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### Abstract

The theory of electron holes is extended into the quantum regime. The Wigner–Poisson system is solved perturbatively based in lowest order on a weak, standing electron hole. Quantum corrections are shown to lower the potential amplitude and to increase the number of deeply trapped electrons. They, hence, tend to bring this extreme non-equilibrium state closer to thermodynamic equilibrium, an effect which can be attributed to the tunneling of particles in this mixed state system.

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Quantum plasmas [1] have recently attracted considerable attention. Non-ideal, dense plasmas generated, e.g., in the ultraintense laser–solid interaction certainly belong to this category. However, also ideal plasmas—the addressee of this Letter—can exhibit a quantum behavior. One reason is that the miniaturization of today’s micro- and nano-electronic components has reached a level such that the system length becomes comparable with the de Broglie wavelength, in which case tunneling effects are no longer negligible [2]. Also, states in combined traps attained by particles and their anti-particles used to form anti-hydrogen may be modelled by a plasma having quantum features [3]. Other examples can easily be found, and some of

them will be mentioned further below. Hence, classical transport models will unlikely be sufficient to describe the plasma behavior in such devices adequately. Generally the main focus in this respect is the collective charge carrier transport which has been shown [4] to be increased by the presence of coherent structures such as hollow phase space vortices [5]. The latter are also called holes in plasma physics [4–7] because of their associated density depression. They exhibit a remarkable robustness and stability [8].

Before studying the quantum corrections to such holes let us review some further dilute many particle systems and related disciplines that reveal quantum aspects. Charged-particle beams in particle accelerators are typically dilute systems, so quantum effects are usually disregarded. However, a spectrum of phenomena, which recently became more and more important, reveals the existence of several quantum aspects of beam physics connecting the physics of particle accel-

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erator with the frontiers of several disciplines, such as (for instance) plasma physics, radiation beam physics, astrophysics, mesoscopic and condensate physics [9, 10]. Most of these phenomena introduce a sort of quantum correction to the leading classical behavior of the system. For example, quantum excitation [11] plays a role for the long-term stability of longitudinal electron beam dynamics in the high-energy circular accelerating machines while the Sokolov–Ternov effect [12] of spin polarization of electron and proton beams is a manifestation, at the macroscopic level, of the single quantum nature of the beam particles. Numerical phase space investigations based on tracking with the quantum map have shown that quantum corrections can substantially affect the particle beam trajectories in the vicinity of the separatrix [13].

Recently quantumlike methodologies [14] have been applied to a number of classical physical situations, in which  $\hbar$  is replaced by another characteristic parameter of the problem considered. For instance, they have been applied to accelerator physics [15], to plasma physics [16], to surface gravity wave physics [17] and to non-linear optics [18] in an attempt to describe linear and non-linear problems of the dynamics of beams and large amplitude wavepackets.

In principle, all these problems can be formulated, in the configuration space, in terms of a system of Zacharov equations, i.e., non-linear Schrödinger-like equation coupled with one (or more) equation(s) taking into account the reaction of the environment. The corresponding phase space description is the one provided by the Wigner–Moyal quasidistribution [19] whose evolution equation, the von Neumann equation, plays the role of a kinetic-like equation associated with the system.

Analytically, the framework under which Wigner–Moyal quasidistributions have been mostly considered so far is that of particles interacting with a given external, e.g., parabolic potential to analyze coherent and squeezed states. Furthermore, a quantum-like phase space analysis of a paraxial-charged-particle beam transport, travelling through a quadrupole-like device with small sextupole and octupole aberrations, has been carried out showing a satisfactory agreement with the results of the standard tracking simulations [22] and, consequently, the suitability of using the quantum phase-space formalism in particle accelerators. This has been done within the framework of the

thermal wave model [15]. Quantum-like corrections involved in the von Neumann equation have been discussed for paraxial beams of both particles and radiation and compared with the standard classical description [23].

Particles in quantum plasmas moving in their own, self-consistent potential, on the other hand, have not been given much attention so far. An exception are self-consistent but linearized solutions of the Wigner–Poisson system, dealing with quantum corrections to Landau damping of Langmuir waves [20] or to the two-stream instability by means of the Nyquist method [24], and the self-consistent linearized solution of the Wigner–Moyal kinetic-like equation for Langmuir wavepackets coupled with the ion-acoustic wave equation [25]. In particular, the Wigner–Moyal kinetic-like description is suitable for describing the Benjamin–Feir instability (modulational instability) as well as predicting the stabilizing effect of a sort of Landau damping. It is well known that the latter cannot be shown in configuration space, where the system is usually described by the Zakharov equations. By using the pure state formalism, a Landau-type damping has been shown for the longitudinal dynamics of both charged-particle coasting beams and e.m. wave-trains in high-energy circular accelerators and non-linear media, respectively [26]. A similar approach has been extended (mixed state formalism) to ensembles of partially-incoherent waves in different physical situations [25,27,28].

The aim of this Letter is to describe an unmagnetized electron–ion plasma, in which, on the basis of the experimental evidences as well as on theoretical and numerical investigations mentioned above, the quantum nature of the particles is not disregarded. However, it is taken into account only as a weak (perturbative) effect in comparison to the leading classical behavior of the system. Together with the weak quantum effect, we take into account the usual classical electrostatic collective plasma effects coming from the standard meanfield approximation of the coulombian interaction, in such a way that our system is described by a set of coupled equation comprising the von Neumann equation for the Wigner–Moyal quasidistribution and Poisson’s equation. Hereafter, we will refer to this system of equations as WP-system (Wigner–Poisson system). Our goal is to find a self-consistent solution of the WP-system to the lowest order of the

quantum correction. Before formulating our problem in detail, it is worth mentioning some further considerations starting from the classical case.

Classically, electron and ion holes are non-linear, stationary solutions of the Vlasov–Poisson system (VP-system) being omnipresent structures in many driven, collisionless plasmas. They are found in one or other variant in the laboratory [7,29,30], in particle accelerators [31,32], in the laser–plasma interaction [33] and in the extraterrestrial space [34]. Analytical solutions, applicable to kinetic hole structures found in particle accelerators, have been presented in [35].

Generally speaking, the fundamental role of holes arises from the fact that they can non-linearly destabilize a plasma even in linearly stable situations, namely if they possess a negative energy character [36,37]. There is hence accrued interest to extend the studies of holes into the quantum domain, which means that the WP-system has to be employed instead.

In recent numerical studies [38] a multistream model for a current-driven quantum plasma has been applied. Signatures of coherent hole structures appear in the simulation of a statistical mixture of many pure states, with each wave function obeying the Schrödinger–Poisson system. That this system is equivalent to the mixed state WP-system has been shown in [39]. To the best of our knowledge, an analytic self-consistent non-linear solution of the WP-system is still missing in the literature.

In this Letter we present a first rigorous non-linear self-consistent solution of the WP-system assuming weak non-linearity and proximity to the classical VP-system. First we shall refer to the classical e-hole and then study quantum corrections, for which a self-consistent solution is derived.

We are first investigating a standing, classical e-hole which is the simplest inhomogeneous stationary solution of the VP-system for a plasma with immobile ions:

$$[v\partial_x + \phi'(x)\partial_v]f(x, v) = 0, \quad (1a)$$

$$\phi'' = \int dv f(x, v) - 1 := n_e(x) - 1, \quad (1b)$$

where  $f(x, v)$  is the distribution function of electrons in phase space. Here space, velocity and density are normalized by the electron Debye length  $\lambda_D = (kT_e/4\pi n_0 e^2)^{1/2}$ , the electron thermal velocity  $v_{th} = (kT_e/m_e)^{1/2}$  and the electron density,  $n_0$ , where

$m_e$  is the mass of the electrons and  $T_e$  is their temperature. These normalization quantities are defined by the underlying homogeneous thermal equilibrium state which is described by a Maxwellian  $f_M(x, v) = \frac{1}{\sqrt{2\pi}} \exp(-v^2/2)$ . Referring to the potential (or alternative) method [40], being the most appropriate solution method [41], we solve (1a) by

$$f(x, v) = \frac{1}{\sqrt{2\pi}} \left[ \exp(-E)\theta(E) + \exp(-\beta E)\theta(-E) \right], \quad (2)$$

where  $E = v^2/2 - \phi$  represents the single electron energy and  $\theta$  is the step function. The separatrix  $E = 0$  in phase space distinguishes the free ( $E > 0$ ) from the trapped ( $E < 0$ ) electron population. Note that  $f$  in (2), reduces to  $f_M$  as  $\phi \rightarrow 0$ . The electron density in the weak amplitude regime can be written as a half power expansion of  $\phi$  [5,40],

$$n_e(\phi) = 1 + \phi - \frac{4(1-\beta)}{3\sqrt{\pi}}\phi^{3/2} + O(\phi^2). \quad (3)$$

Defining the pseudo-potential as  $-V'(\phi) := n_e(\phi) - 1$ , we find from (3) with  $V(0) = 0$

$$-V(\phi) = \frac{\phi^2}{2} \left( 1 - \frac{16(1-\beta)}{15\sqrt{\pi}}\sqrt{\phi} \right). \quad (4)$$

It has to fulfill two necessary conditions: (a)  $V(\phi) < 0$  in  $0 < \phi < \psi$  and (b)  $V(\psi) = 0$ , where  $\psi$  is the amplitude of the self-consistent potential, which is assumed to be small,  $\psi \ll 1$ . From (b) we arrive at  $-\beta = 15\sqrt{\pi}/16\sqrt{\psi} - 1 \approx 15\sqrt{\pi}/16\sqrt{\psi} \ll 1$ . Therefore,  $\beta$  has to be a large negative number, corresponding to a depletion of the distribution in the trapped particle range. On the other hand, Eq. (4) allows us to integrate Poisson's equation (1b), and we obtain the bell-shaped electrostatic potential:

$$\phi(x) = \psi \operatorname{sech}^4\left(\frac{x}{4}\right). \quad (5)$$

Note that other electrostatic structures such as propagating electron holes, ion holes or periodic non-linear waves (cnoidal waves) can also be found by appropriate extensions of this method [5,6,40,42].

To study quantum corrections to hole equilibria, we start with Wigner's quasidistribution which satisfies the time independent von Neumann–Weyl–Wigner–

Moyal equation [19]:

$$v\partial_x f + \frac{1}{i\varepsilon} \left[ \phi \left( x + \frac{i\varepsilon}{2} \partial_v \right) - \phi \left( x - \frac{i\varepsilon}{2} \partial_v \right) \right] f = 0, \quad (6)$$

where  $\varepsilon$  is the dimensionless Planck's constant  $\varepsilon = \frac{\hbar}{m_e v_{th} \lambda_D} = \lambda_{dB}/\lambda_D$ ,  $\lambda_{dB}$  and  $\lambda_D$  being thereby the thermal de Broglie length and the Debye screening length. In (6) the uncertainty of the particle position is reflected by the potential operator  $\phi(x \pm \frac{i\varepsilon}{2} \partial_v)$ . The validity of (6), as was laid down in the literature [19,20], assumes the absence of degeneracy and of binary collisions, which implies that  $\lambda_{dB} < \lambda_n < \lambda_D$ , where  $\lambda_n = n^{-1/3}$  is the mean particle distance.

In the quasiclassical approximation, when quantum effects appear only as a correction,  $\varepsilon \ll 1$  and we can perform a power expansion in  $\varepsilon$  of the potential operator and get

$$v\partial_x f + \phi'(x)\partial_v f - \frac{\varepsilon^2}{3!4} \phi'''(x)\partial_v^3 f = 0, \quad (7)$$

where terms of  $O(\varepsilon^4)$  were neglected. It is valid as long as the correction to the Vlasov terms dominates over the lowest order collision term, which is  $O((\lambda_n/\lambda_D)^3)$  [21]. The range of validity of the present approach is, hence, restricted by the following constraints:

$$\lambda_{dB} < \lambda_n < (\lambda_{dB}^2 \lambda_D)^{1/3} < \lambda_D, \quad (8)$$

which are met in sufficiently hot and dense plasmas such as those found in the intense laser/particle beam–solid interaction and possibly in the interior of giant planets.

As we keep terms up to  $O(\varepsilon^2)$ , we will look for corrections of the same order in the potential and in the distribution function,  $f = f_0 + \varepsilon^2 f_1$ ,  $\phi = \phi_0 + \varepsilon^2 \phi_1$ , with  $f_0$  and  $\phi_0$  representing now (2) and (5), respectively. Inserting this ansatz into (7) and (1b) neglecting again terms of  $O(\varepsilon^4)$  we find

$$[v\partial_x + \phi'_0(x)\partial_v]f_1 = -\phi'_1(x)\partial_v f_0 + \frac{1}{3!4} \phi_0'''(x)\partial_v^3 f_0, \quad (9a)$$

$$\partial_x^2 \phi_1(x) = \int dv f_1. \quad (9b)$$

By defining  $g(x, v) := f_1 + \phi_1 \partial_E f_0$ , we can reduce Eqs. (9a) and (9b) to the somewhat simpler system

$$[v\partial_x + \phi'_0(x)\partial_v]g = \frac{1}{3!4} \phi_0'''(x)\partial_v^3 f_0 =: h(v, x), \quad (10a)$$

$$\phi_1''(x) + V''(\phi_0)\phi_1(x) = \int_{-\infty}^{+\infty} dv g(x, v). \quad (10b)$$

Now it is convenient to switch into a new set of variables defined by  $\xi = x$ ,  $E = v^2/2 - \phi_0(x)$ ,  $\sigma = \text{sg}(v)$  and rewrite  $h(x, v) = H(\xi, E, \sigma)$ ,  $g(x, v) = G(\xi, E, \sigma)$ . With these variables, (10a) becomes  $\partial_\xi G(\xi, E, \sigma) = H(\xi, E, \sigma)/v(\xi, E, \sigma)$ , whose general solution is

$$G(\xi, E, \sigma) = G(0, E, \sigma) + \int_0^\xi d\xi' \frac{H(\xi', E, \sigma)}{v(\xi', E, \sigma)}, \quad (11)$$

where  $v(\xi, E, \sigma) = \sigma\sqrt{2[E + \phi_0(\xi)]}$ . Therefore, in order to find  $G$  we only have to integrate  $H(\xi, E, \sigma)/v(\xi, E, \sigma)$  along the classical particle trajectories given by  $E = \text{const}$ . In this expression we have chosen the lower integration limit as  $\xi = 0$  because this is the only point which is reached by all trajectories (see below). Note that a trapped particle will move along a closed, bounded trajectory around the origin in phase space.

Now we need to replace  $H(\xi, E, \sigma)$  by its full expression. Denoting  $f$  in Eq. (2) as  $f_0(E)$  we get by differentiation

$$\begin{aligned} \partial_v^3 f_0 &= \frac{1}{\sqrt{2\pi}} v \{ [3 - 2(E + \phi_0)] e^{-E} \theta(E) \\ &\quad + \beta^2 [3 - 2\beta(E + \phi_0)] e^{-\beta E} \theta(-E) \\ &\quad - [3(1 - \beta) - 2\phi_0(1 - \beta^2)] \delta(E) \\ &\quad - 2(E + \phi_0)(1 - \beta) \delta'(E) \} \\ &=: \frac{1}{\sqrt{2\pi}} v(\xi, E, \sigma) \Omega(\xi, E). \end{aligned} \quad (12)$$

For positive energies, we can follow the trajectories up to any  $\xi$  in (11) and, assuming that the correction vanishes at  $\xi \rightarrow \pm\infty$ , we arrive at

$$\begin{aligned} G(0, E, \sigma) &= \int_{-\infty}^0 d\xi' \frac{H(\xi', E, \sigma)}{v(\xi', E, \sigma)} \\ &= \frac{1}{3!4\sqrt{2\pi}} \int_{-\infty}^0 d\xi' \phi_0'''(\xi') \Omega(\xi', E). \end{aligned} \quad (13)$$

Note that this expression does no longer depend on  $\sigma$ . For negative energies  $G(0, E, \sigma)$  is not determined by such a procedure but, due to the symmetry of the problem we can assume that it will also be  $\sigma$ -independent. On the other side, we can always extend the integration of (11) to  $-\infty$  for negative energies also as long as we change the integration constant. Therefore we have, for any  $E$ ,

$$\begin{aligned} G(\xi, E, \sigma) &= G(\xi, E) \\ &:= \frac{1}{3!4\sqrt{2\pi}} \left[ G_0(E) + \int_{-\infty}^{\xi} d\xi' \phi_0'''(\xi') \Omega(\xi', E) \right], \end{aligned} \quad (14)$$

with  $G_0(E) = 0$  for  $E > 0$ .

Note that  $\partial_E f_0$  is discontinuous at  $E = 0$ . Therefore,  $G(\xi, E)$  does not have a definite value at the separatrix. Our approach will be to solve (10) for positive and negative energies separately and then put both solutions together imposing the continuity of  $f_1$  at the separatrix.

In order to integrate (14), we consider these two different cases:

1. For  $E > 0$  we have  $G_0(E) = 0$  and  $\Omega(\xi, E) = [3 - 2(E + \phi_0(\xi))]e^{-E}$ . The integral (14) can be performed analytically to yield

$$G(\xi, E) = \frac{1}{3!4\sqrt{2\pi}} \left[ \phi_0'(\xi)^2 + (3 - 2E - 2\phi_0(\xi))\phi_0''(\xi) \right] e^{-E}.$$

2. If  $E < 0$  we must take  $G_0(E)$  into account. In this case  $\Omega(\xi, E) = \beta^2 [3 - 2(E + \phi_0(\xi))\beta] e^{-\beta E}$  and (14) reads

$$\begin{aligned} G(\xi, E) &= \frac{1}{3!4\sqrt{2\pi}} \\ &\times \{ G_0(E) + [\beta\phi_0'(\xi)^2 + (3 - 2\beta E - 2\beta\phi_0(\xi))\phi_0''(\xi)]\beta^2 e^{-\beta E} \}. \end{aligned} \quad (15)$$

The continuity of  $f_1$  is now imposed to determine  $G_0(E)$ . As  $f_1 = g - \phi_1 \partial_E f_0$ , the discontinuity of  $g(x, v) = G(\xi, E)$ , namely  $\Delta G := G(\xi, 0^+) - G(\xi, 0^-)$  should be equal to  $\phi_1 \Delta(\partial_E f_0)$ . Since it holds

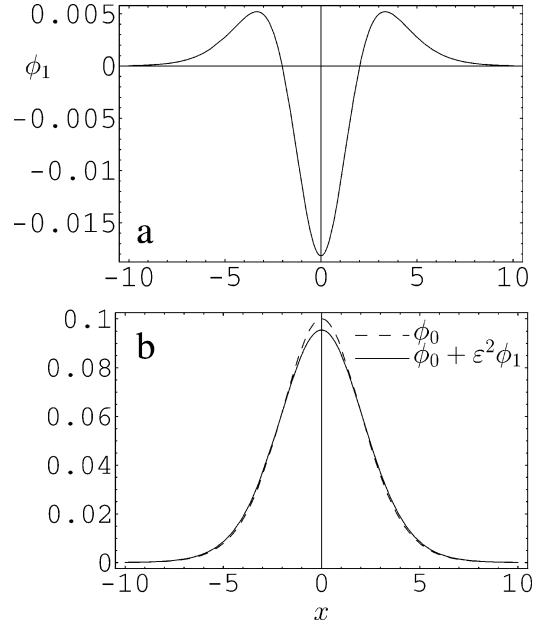


Fig. 1. (a) Correction to the potential for  $\psi = 0.1$ ; (b) the classical and the quantum corrected potential.

$\Delta(\partial_E f_0) = (\beta - 1)/\sqrt{2\pi}$ , we get

$$\begin{aligned} \Delta G &= \frac{1}{3!4\sqrt{2\pi}} \{ \phi_0'(\xi)^2 + (3 - 2\phi_0(\xi))\phi_0''(\xi) \\ &\quad - \beta^2 [\beta\phi_0'(\xi)^2 + (3 - 2\beta\phi_0(\xi))\phi_0''(\xi)] \\ &\quad - G_0(0) \}. \end{aligned} \quad (16)$$

Then we can find  $\phi_1(\xi)$  as  $\phi_1(\xi) = \Delta G / \Delta(\partial_E f_0) = \sqrt{2\pi} \Delta G / (\beta - 1)$  with  $\Delta G$  given by (16). Moreover, as we impose  $\phi_1(\pm\infty) = 0$ , we know that  $G_0(0) = 0$ . Hence, we obtain  $\phi_1(\xi)$ . Fig. 1(a) shows  $\phi_1(\xi)$  for  $\psi = 0.1$  and Fig. 1(b) represents the corrected potential. For reference, also the unperturbed potential is drawn. We see that the potential experiences a reduction as a result of quantum correction.

To determine  $G_0(E)$  for all negative energies we go back to (10b). As we have already determined  $\phi_1(x)$ , the left-hand side is now given. It is convenient to write it in terms of  $\phi_0$ . To do this we note that all derivatives of  $\phi_0$  can be expressed by  $\phi_0$  itself via  $\phi_0'(\xi) = -\phi_0(\xi)(1 - \sqrt{\phi_0(\xi)/\psi})^{1/2}$ ,  $\phi_0''(\xi) = \phi_0(\xi)(1 - 5\sqrt{\phi_0(\xi)/\psi}/4)$  and also  $V''(\phi_0) = -(1 - 15\sqrt{\phi_0/\psi}/8)$ . Inserting these expressions into (10b) we find an expression for its left-hand side as a

function of  $\phi_0$  which we call  $L(\phi_0)$ . An explicit form of this function is too long to be included in this Letter.

On the other hand, the right-hand side of (10b) can be written as

$$\begin{aligned} \int_{-\infty}^{+\infty} dv g(x, v) &= \sum_{\sigma} \sigma \int_{-\phi_0(\xi)}^{\infty} dE \frac{G(\xi, E)}{v(\xi, E, \sigma)} \\ &= 2 \int_{-\phi_0(\xi)}^{\infty} dE \frac{G(\xi, E)}{v(\xi, E, 1)}. \end{aligned} \quad (17)$$

Making use of (14) we can reduce (17) to

$$\begin{aligned} \int_{-\infty}^{+\infty} dv g(x, v) \\ = R(\phi_0) + \frac{1}{3!2\sqrt{2\pi}} \int_{-\phi_0(\xi)}^0 dE \frac{G_0(E)}{v(\xi, E, 1)}, \end{aligned} \quad (18)$$

where  $R(\phi_0)$  represents a known function that can be obtained analytically.

To perform the remaining integral of (18), we make a half power expansion of  $G_0(E)$ :

$$\begin{aligned} \frac{1}{3!2\sqrt{2\pi}} G_0(E) \\ = a_{1/2}|E|^{1/2} + a_1|E| + a_{3/2}|E|^{3/2} + \dots \end{aligned} \quad (19)$$

With this ansatz, we have

$$\begin{aligned} \frac{1}{3!2\sqrt{2\pi}} \int_{-\phi_0(\xi)}^0 dE \frac{G_0(E)}{v(\xi, E, 1)} \\ = \sqrt{\phi_0} \sqrt{\frac{\pi}{2}} \sum_{n=1}^{\infty} \frac{\Gamma(1 + \frac{n}{2})}{\Gamma(\frac{3}{2} + \frac{n}{2})} a_{n/2} \phi_0^{n/2}. \end{aligned} \quad (20)$$

And we can finally reduce (10b) to

$$\begin{aligned} L(\phi_0) &= R(\phi_0) \\ &+ \sqrt{\phi_0} \sqrt{\frac{\pi}{2}} \sum_{n=1}^{\infty} \frac{\Gamma(1 + \frac{n}{2})}{\Gamma(\frac{3}{2} + \frac{n}{2})} a_{n/2} \phi_0^{n/2}. \end{aligned} \quad (21)$$

Therefore, if we define  $\rho(t) := \frac{1}{t}(L(t^2) - R(t^2))$ , we can find all  $a_{n/2}$  as

$$a_{n/2} = \frac{1}{n!} \sqrt{\frac{2}{\pi}} \frac{\Gamma(\frac{3}{2} + \frac{n}{2})}{\Gamma(1 + \frac{n}{2})} \left. \frac{d^n \rho(t)}{dt^n} \right|_{t=0}. \quad (22)$$

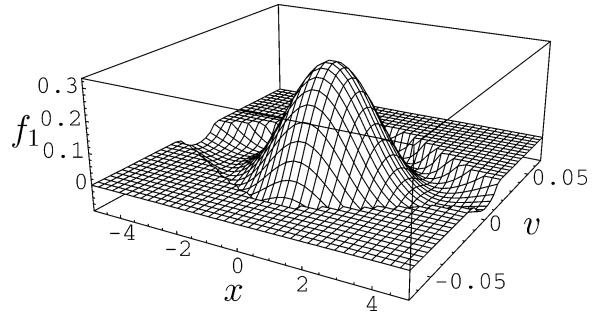


Fig. 2. Correction of the distribution function  $f_1(x, v)$ .

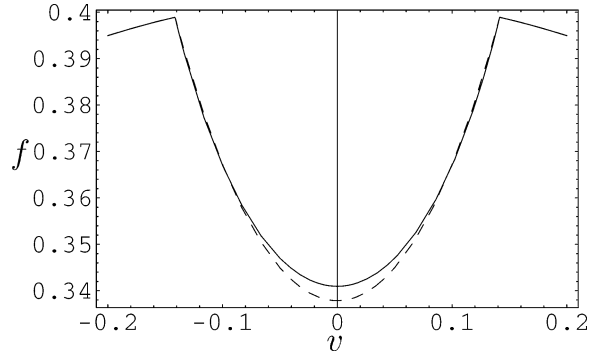


Fig. 3. Corrected distribution function  $f = f_0 + \varepsilon^2 f_1$  at  $x = 0$ , for  $\psi = 0.01$ ,  $\varepsilon = 0.1$ . The dashed line represents the original (unperturbed) distribution function.

With this expression for  $a_{n/2}$  we can sum  $G_0(E)$  and then find  $G(\xi, E)$  and  $f_1$ . The correction of the distribution function,  $f_1(x, v)$  is plotted in Fig. 2, while the final, corrected distribution function  $f = f_0 + \varepsilon^2 f_1$  is represented at fixed  $x$  in Fig. 3.

We clearly recognize a partial filling of the phase space within the separatrix being maximum at the hole center. An interpretation may be given in terms of refraction or tunneling: in the classical solution nearby its separatrix, the region of untrapped electrons is populated stronger than that of trapped electrons. In the quantum domain when tunneling becomes effective this gives rise to a net influx of particles resulting in a less dilute distribution of trapped electrons. This interpretation of the collective particle behavior, found analytically, matches well with the numerical findings of Ref. [13], according to which the quantum corrections affect the particle trajectories in the vicinity of the separatrix (see also Ref. [22]).

We, therefore, conclude that the overall effect of a quantum correction to a classical e-hole is the ten-

dency of the system to reduce the coherent excitation by both a diminution of the amplitude and a partial filling of the trapped particle region by refraction (tunneling), bringing the system closer to the thermal state.

Open questions are how these semiclassical corrections are modified in case of finite amplitudes  $\Psi \gtrsim O(1)$ , of finite quantum corrections  $\varepsilon \gtrsim O(1)$ , of hole propagation  $v_0 > 0$  and of non-locality of structures such as periodic wave trains.

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